5 January 2004
revised 11 January 2004

INTRODUCTION TO MATHEMATICAL MODELLING
LECTURE 1: OVERVIEW

David A. Meyer

Project in Geometry and Physics, Department of Mathematics
University of California/San Diego, La Jolla, CA 92093-0112
http://math.ucsd.edu/~dmeyer/; dmeyer@math.ucsd.edu

Office hours: AP&M 7256,T 12:30–1:30, F 1:00–2:00, or by appointment

Introduction

The first upper division mathematics course many students take is some sort of Introduction to Mathematical Reasoning class, in which they are supposed to learn how to prove mathematical theorems. In that course the answers to questions like, "What is this good for?", are: "These are the structures of logical arguments." or "It's a technique we can use to prove many theorems." or "It's a cool argument/result." or "We're going to use it to prove something else, later.". Many mathematicians would say that this is what mathematicians do, and what mathematics is about—proving theorems.

This class is at the other extreme. Our goal is to learn how to *use* mathematics to understand/explain/describe/predict aspects of the real world. This is what scientists and engineers do—they use mathematics as a language that is more precise, clearer, and more useful for their purposes than 'natural' language. Despite the judgement asserted at the end of the previous paragraph, this is the context in which the bulk of mathematical language is used.

There is perhaps an analogy with natural language: A small fraction is used for literary purposes, *e.g.*, poetry, where a substantial emphasis is on language for language's sake, while by far the majority of language is more utilitarian, *e.g.*, conversation, or newspaper writing. Just as many mathematicians would emphasize the importance of proofs, many more literature professors study and teach poetry than newspaper articles.

Having just spent part of the holidays with my 4 nieces and nephew, however, it seems to me that utilitarian language is developmentally, historically and logically most likely prior to, and at a least in concert with, poetic language. The same is true of mathematics, although this is perhaps not so clear from the way we learn it presently. Applications drove the early creation of mathematics: counting, measuring land, navigation, *etc.*, and pure mathematics continues to develop in tandem with applications: low-dimensional topology and gauge theory, algebraic geometry and string theory, graph theory and analysis of the Internet, *etc.*

That is what we will do in this class—we will try to describe and analyze some real world phenomena mathematically, which may require learning some new mathematics, and even proving a few theorems along the way.

> The prerequisites for this course are calculus and either linear algebra or ordinary differential equations. I will explain parts of probability and statistics, analysis, and combinatorics as we need them; some of this material may be new to some students. It will be useful to be fluent in Matlab or Mathematica, or in some general purpose programming language.

Topics

So which phenomena will we consider? There are many, many aspects of the real world that we might try to understand mathematically. Among the sciences, physics is the most mathematicized. The successes of mathematical physics will strongly influence how we approach real world modelling in this course, and which topics we cover. For example, I plan to spend some time discussing the heat equation. But one can take physics courses to learn Maxwell's equations for electromagnetism, or Einstein's general theory of relativity, so this course will be focussed elsewhere.

In the social sciences, mathematical descriptions date back hundreds of years: by Graunt in the 1600s, by Condorcet in the 1700s, by Quetelet in the 1800s (to list only a few representative names), and throughout economics in the 1900s. Nevertheless, there has been so much less success than in the physical sciences that Asimov could write a series of six science fiction novels (*The Foundation Series* [1]) premised on the development of a mathematical theory of social evolution. This means that in this class we can discuss interesting social science phenomena using relatively elementary mathematics, without overlapping with courses in other departments—and we even have opportunities to develop novel models of our own.

In particular, I plan to address topics in urban/regional studies, transportation and communication systems, migration, and the Internet. The course will only work, however, with substantial class participation, so we will also spend time on topics suggested by the students. Since the course will continue as MATH 111B in the Spring quarter, there is no rush to complete any given syllabus this quarter.

Requirements

I expect interest and enthusiasm from the students in this class. Grades will be determined as follows:

[30%] class participation—often describing homework solutions
homework—only sometimes to hand in

[70%] project—a mathematical model of something, either part of a joint class project on a topic that will unfold as the course continues, or a separate individual or small group project on some other topic

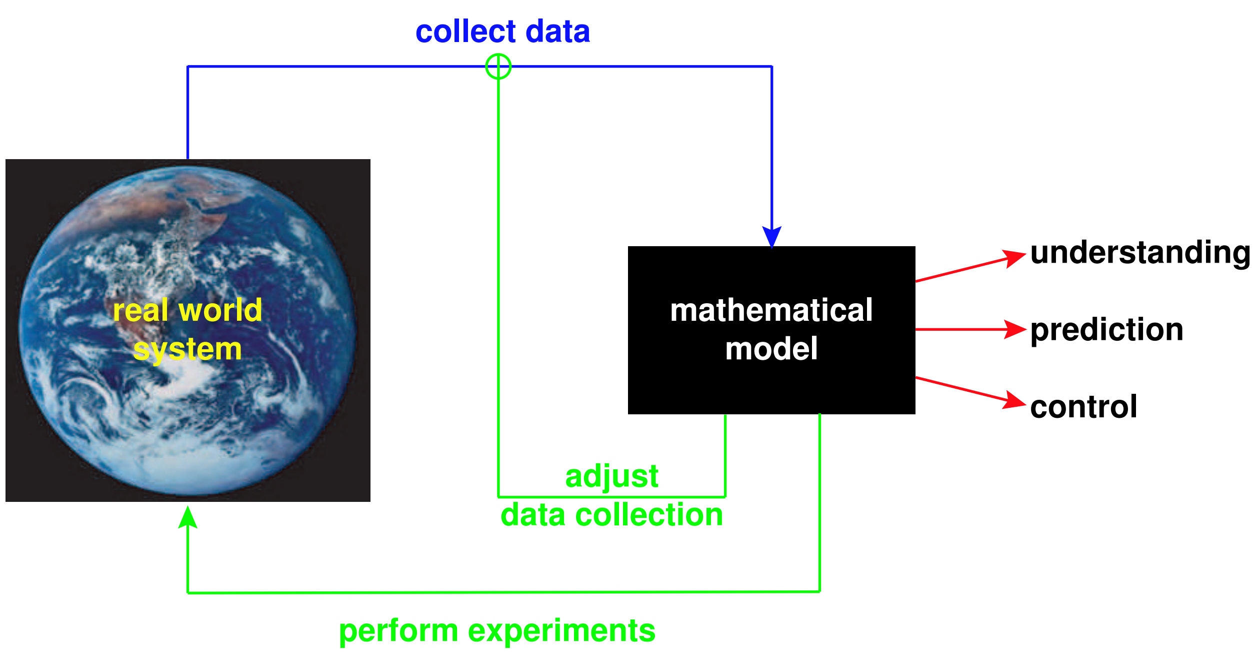
[30%] short written preliminary description due in Week 4

[40%] short presentation and longer written final description in Weeks 8–10

I will make these requirements more precise as the course evolves.

Modelling

Putting aside philosophical issues, let us take the position that there is a real world (if there were sufficient interest, I might be willing to spend a lecture on some of these philosophical issues), aspects of which we would like to model. By this I mean, roughly, construct some mathematical representation that enhances our understanding, allows us to make predictions, and possibly informs us how to control the part of the real world (the *system*) we care about. The figure below illustrates the process of observing the system, collecting the consequent data, feeding that data into the mathematical model, and possibly using analysis of the model to adjust the data collection, including doing experiments



on the system. We will discuss what constitutes a mathematical model in greater detail later. In the meantime:

Homework: Read E. A. Bender, *An Introduction to Mathematical Modeling* (Mineola, NY: Dover 2000), Chapter 1.

Data collection

It is impossible to make any model of the real world without knowing something about the system that we are trying to model. That knowledge may be tacit, but in principle it must come from observations, *i.e.*, data collection. And for complicated systems there are certainly large quantities of data that one would like to incorporate into a model and understand. Without intending to over-emphasize this aspect of mathematical modelling, and certainly not intending to turn this into a statistics course, let us nevertheless begin by collecting data about a specific aspect of the real world.

References

[1] I. Asimov, *Prelude to Foundation*, *Foundation*, *Foundation and Empire*, *Second Foundation*, *Foundation's Edge*, *Forward the Foundation* (New York: Doubleday 1988, 1951, 1952, 1953, 1982, 1993).