Consider a random walk on $\mathbb{Z}/N\mathbb{Z}$, $N \in \mathbb{N}$, with a “sink” at $N - 1$, i.e., with a transition probability matrix

$$
P = \begin{pmatrix}
1/2 & 1/4 \\
1/4 & 1/2 & 1/4 \\
& 1/4 & 1/2 \\
& & 1/4 & \ddots & 1/4 \\
& & & 1/2 \\
1/4 & & & & 1/4 & 1
\end{pmatrix}.
$$

1. Write code to start at a uniformly random state and evolve according to this random walk. For $N \in \{10, 20, 30\}$, show histograms of the position after $T \in \{100, 500, 1000\}$ steps.

2. a. Find a stationary distribution for the stochastic process defined by $P$.

b. Let $\theta = \pi/N$. Show that the vector $v$ with

$$
v_s = \begin{cases}
-\sin\left((s + 1)\theta\right) & \text{if } s \in \{0, 1, \ldots, N - 2\} \\
\sin \theta/(1 - \cos \theta) & \text{if } s = N - 1
\end{cases}
$$

is an eigenvector of $P$ and find its eigenvalue.

c. Use your answers for questions 2.a. and 2.b. to explain your results in problem 1, specifically how close each histogram is to the stationary distribution.