Random Walk Algorithms: Homework 6

You can answer the following questions either analytically, or by writing and running code.

1. Consider a game A which consists of flipping a coin with probability $1/2$ of landing head up, in which case you win $1$; if it lands tail up you lose $1$. Thus, if $W_t$ is your wealth after playing $t$ times, \( \{W_t\} \) is a random walk on $\mathbb{Z}$. If $W_0 = 0$, what is $E[W_{100}]$?

2. Now consider a game B which has two coins, $B_1$ and $B_2$. The probability of $B_1$ landing head up is $1/10$ and the probability of $B_2$ landing head up is $3/4$. On play $t$, if $W_{t-1} \equiv 0 \pmod{3}$ you must flip $B_1$; otherwise you must flip $B_2$. Again you win $1$ if the coin you flip lands head up; otherwise you lose $1$. In this case, \( \{W_t\} \) is an inhomogeneous random walk on $\mathbb{Z}$. If $W_0 = 0$, what is $E[W_{100}]$?

3. Finally, suppose you play these games in the order AABB. If $W_0 = 0$, what is $E[W_{100}]$? Compare your result with those in problems 1 and 2.