

180A PRACTICE MIDTERM 2

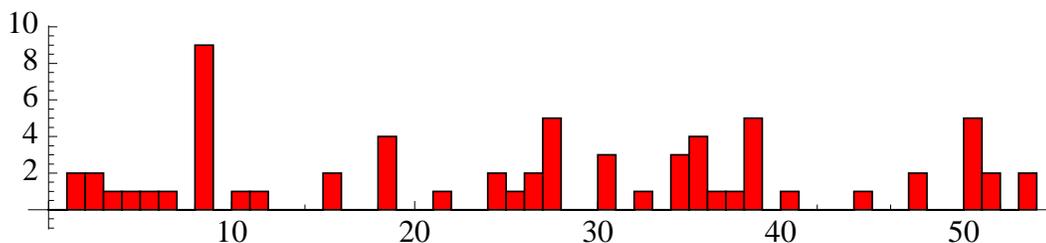
I think these problems are representative, but you may find the actual midterm either more or less difficult than this practice midterm. You may use a calculator.

1. Ecologists want to estimate the number of Northern pike (a non-native invasive fish species) in Lake Davis. They set nets at multiple locations in the lake one day, and catch 1000 Northern pike. Each is tagged and released. The next day they reset the nets and catch 1500 Northern pike, of which 20 have tags from the previous day.
 - a. (10 points) If there are N Northern pike in Lake Davis, what is the probability of catching 20 out of 1500 with tags?
 - b. (10 points) How many Northern pike would you estimate there are in Lake Davis?
 - c. (5 points) What assumptions are you making in parts (a) and (b)?
2. Let A_1, A_2, B_1 and B_2 be random variables taking values ± 1 . Without knowing anything about the joint distribution of these four random variables:
 - a. (10 points) What is the largest and the smallest possible value for $E[A_1 B_1]$?
 - b. (15 points) What is the largest and the smallest possible value for $E[A_1 B_1 + A_2 B_1 + A_1 B_2 - A_2 B_2]$?

3. Let X and Y be random variables taking values in $\{0, 1\}$. Suppose their joint probability distribution is

$$P(X = x, Y = y) = \begin{cases} 1/3 & \text{if } x = y; \\ 1/6 & \text{otherwise.} \end{cases}$$

- a. (10 points) Find the marginal distributions.
 - b. (5 points) What is $E[X]$?
 - c. (10 points) What is the covariance, $\text{Cov}[X, Y]$?
4. The graph shows the number of events per day for the first 54 days of this year. There are 67 events.



- a. (5 points) Let C be a random variable with probability distribution $P(C = c) = (\text{number of days on which there are } c \text{ events})/54$. What is $E[C]$?
- b. (10 points) What is $\text{Var}[C]$?
- c. (10 points) Do you think these events are a Poisson scatter?
- d. (5 points, extra credit) What are these events?