1.4 \quad P(A) = 0.4, \quad P(B) = 0.7, \quad \text{since}

\[ P(AB) = P(A) + P(B) - P(A \cup B) = 1.1 - P(A \cup B), \]

and \( P(A \cup B) \leq 1 \).

This gives \( P(AB) \geq 1.1 - 1 = 0.1 \) \hspace{2cm} (11)

Then, \( AB = A \cap B \cap C \setminus A \), so

\[ P(AB) \leq P(A) = 0.4. \] \hspace{2cm} (12)

Combining (11) and (12) completes the proof.

1.34

\[ \frac{\text{Area (shaded square)}}{\text{Area (square)}} = \frac{\frac{1}{3} \times \frac{1}{3}}{1} = \frac{1}{9}. \]

1.40

Method 1. Using the hint, we denote

\[ G = \{ \text{exactly 2 balls are green} \} \]
\[ R = \{ \text{exactly 2 balls are red} \} \]
\[ Y = \{ \text{exactly 2 balls are yellow} \} \]
\[ W = \{ \text{exactly 2 balls are white} \} \]

\[ P(G) = P(R) = P(Y) = P(W) = \left( \frac{1}{2} \right)^2 \left( \frac{3}{4} \right)^2 \]

\[ P(GAR) = P(GAY) = P(GAW) = P(RAY) = P(RAW) = P(YAW) = \left( \frac{4}{2} \right) \left( \frac{1}{4} \right)^2 \left( \frac{3}{4} \right)^2 \]

\[ P(GARAY) = P(GARAW) = P(GAYAW) = P(RAYAW) = 0 \]

\[ P(GARAYNW) = 0 \]

By inclusion-exclusion formula:

\[
P(GURUYUW) = P(G) + P(R) + P(Y) + P(W) - \left[ P(GAR) + P(GAW) + P(GAY) + P(RAY) + P(RAW) + P(YAW) \right]
\]

\[
= 4 \cdot \left( \frac{1}{2} \right) \left( \frac{4}{2} \right)^2 \left( \frac{3}{4} \right)^2 - 6 \cdot \left( \frac{1}{2} \right) \left( \frac{1}{4} \right)^2 \left( \frac{3}{4} \right)^2 = \frac{45}{64}
\]

Method 2:

\[ P(\{ \text{at least one color is repeated twice} \}) \]

\[ = P(\{ \text{one color is repeated twice} \}) + P(\{ \text{two colors are repeated twice} \}) \]

\[ = \frac{\left( \frac{4}{1} \right) \left( \frac{4}{2} \right) \left( \frac{3}{1} \right) \left( \frac{1}{2} \right) + \left( \frac{4}{2} \right) \left( \frac{1}{2} \right)}{4^4} = \frac{45}{64} \]
2.3 Numbers that have at least 1 digit equal to 5:
5, 15, 25, 35, 45, 50, 51, 52, 53, 54, 55, 56, 57,
58, 59, 65, 75, 85, 95.
Among them, these are divisible by 3:
15, 45, 51, 54, 57, 75.
So, \( P = \frac{6}{19} \).

2.10 Denote \( A = \{ \text{outcome is 4} \} \).
\( B = \{ \text{I pull out the 6-sided die} \} \).
\[
P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A|B) \cdot P(B)}{P(A)}.
\]
\[
P(A \cap B) = \frac{1}{3} \cdot \frac{1}{6}.
\]
\[
P(A) = \frac{1}{3} \cdot \frac{1}{4} + \frac{1}{3} \cdot \frac{1}{6} + \frac{1}{3} \cdot \frac{1}{12}
\]
So, \( P(B|A) = \frac{1}{3} \).

2.16 \( A_1 = \{ H \ldots \} \).
\( A_2 = \{ HT, TH \} \).
\( A_3 = \{ HHT, THT, TTH, HHH \} \).
where "" means it can be either H or T.
\[ P(A_1) = \frac{1}{2}, \quad P(A_2) = \frac{1}{2}, \quad P(A_3) = \frac{1}{2}. \]

It can be checked that

\[ P(A_1 \cap A_2) = P(\{HT, HT\}) = \frac{1}{4} = P(A_1)P(A_2). \]
\[ P(A_1 \cap A_3) = P(\{HT, TH\}) = \frac{1}{4} = P(A_1)P(A_3). \]
\[ P(A_2 \cap A_3) = P(\{TT, TT\}) = \frac{1}{4} = P(A_2)P(A_3). \]
\[ P(A_1 \cap A_2 \cap A_3) = P(\{TT\}) = \frac{1}{8} = P(A_1)P(A_2)P(A_3). \]

So, \( A_1, A_2, A_3 \) are independent.

2.22. (a) \[ P(\{Ann wins\}) = \frac{1}{2}. \]

(b) \[ P(\{Ann's first win happens in the 4th round\}) \]
\[ = \left(\frac{2}{3}\right)^3 \cdot \frac{1}{2} = \frac{8}{27}. \]

(c) \[ P(\{Ann's first win comes after 4th round\}) \]
\[ = P(\{Ann doesn't win for the first 4 rounds\}) \]
\[ = \left(\frac{2}{3}\right)^4 = \frac{16}{81}. \]

2.26. \[ P(AB \cap CP) = P(AB)P(CP) \quad \text{Goal}. \]

RHS = \[ P(AB)P(CP) = P(A)P(B)P(C)P(D) \]
= \[ P(AB \cap CP) = P((AB) \cap (CD)) = P(AB \cap CP) = \text{LHS}. \]
2.46  (a) \( M = \{ \text{ball chosen from A is 2} \} \)

\[ N = \{ \text{two balls numbered 1 are chosen} \} \]

\[
P(M \mid N) = \frac{P(M \cap N)}{P(N)} = \frac{\frac{1}{6} \cdot \frac{1}{12} \cdot \frac{3}{4}}{\frac{1}{6} \cdot \frac{1}{12} \cdot \frac{3}{4} + \frac{1}{6} \cdot \frac{1}{12} \cdot \frac{1}{4} + \frac{5}{6} \cdot \frac{1}{12} \cdot \frac{1}{4}}
\]

\[= \frac{14}{19}.\]

(b) \( M = \{ \text{ball chosen from B is 12} \} \)

\[ N = \{ \text{mean of 3 balls is 7} \} \]

\[
P(M \mid N) = \frac{P(M \cap N)}{P(N)} = \frac{P(\{6, 12, 3\}, \{5, 12, 4\})}{P(\{6, 12, 3\}, \{5, 12, 4\}, \{6, 11, 4\})}
\]

\[= \frac{2}{3}.\]

1.57  (a). To count the number of possible cases.

Case

- All dominoes are vertical  \(1\)
- Two columns are tiled horizontally  \(8\)
- 4 columns are tiled horizontally  \(7\)
- 6 columns are tiled horizontally  \(6\)
- 8 columns are tiled horizontally  \(5\)
So the total number is $1 + \binom{8}{1} + \binom{7}{2} + \binom{6}{3} + \binom{5}{4} = 55$.

$P(\{\text{all dominoes are placed vertically}\}) = \frac{1}{55}$.

(b) Count the number of cases such that the 5th column is a vertical domino:

$P(\{\text{a vertical domino in the middle of the board}\}) = \frac{1 + 6 + 11 + 6 + 1}{55} = \frac{5}{11}$. 