1. [25 points] Let $A$, $B$, and $C$ be events in a probability space $(\Omega, \mathcal{F}, P)$. Suppose $P(A) = P(B) = P(C) = \frac{1}{2}$. What is the smallest possible value for $P(A \cap B) + P(B \cap C) + P(C \cap A)$?

2. A special unfair die has probabilities of rolling $m$ and $n$ whose ratio is $m/n$, for all $m, n \in \{1, 2, 3, 4, 5, 6\}$.
   a. [10 points] Find $P(n)$ for each $n \in \{1, 2, 3, 4, 5, 6\}$.
   b. [10 points] If you roll the die twice, what is the probability that the sum of your two rolls is 7?
   c. [5 points] Is your answer to (b) larger or smaller than what the probability would be if you were rolling a fair die?

3. [25 points] You play the following game with a fair die: Roll the die. If it is $n$, you roll the die $n$ more times. If you roll a second $n$, you win. What is the probability that you win?

4. [25 points] Let $Z = (X, Y)$ be a point chosen uniformly at random in the unit square $[0, 1]^2 = \{(x, y) : 0 \leq x, y \leq 1\}$. Find the cumulative distribution function for the random variable $D =$ distance from $Z$ to the closest point on the boundary of the square, and then find its probability density function.