180A PRACTICE PROBLEMS FOR MIDTERM 1

Please simplify your answers to the extent reasonable without a calculator. Show your work. Explain your answers, concisely.

1. Let $A$ and $B$ be events in a probability space $(\Omega, \mathcal{F}, P)$.
   a. Suppose $P(A) + P(B) > 1$. Making no further assumptions on $A$ and $B$, prove that $A \cap B \neq \emptyset$.
   b. Suppose $A \cap B = \emptyset$. If $A$ and $B$ are independent, what can you say about $P(A)$ and $P(B)$?
   c. Suppose $P(A) = 1/2$ and $P(B) = 4/5$. What are the smallest and largest possible values for $P(A \cap B)$?

2. You are playing a game in which you roll 2 dice. If the sum of the two numbers showing is greater than or equal to 10, you win.
   a. What is the probability that you win the first three times you play?
   b. What is the probability that you win exactly three times out of the first five times you play?
   c. What is the probability that the first game you win is before the tenth game, but after the fifth?

3. A box contains three coins, two of which are fair and third of which has $P(\text{heads}) = 3/4$. A coin is chosen randomly from the box and flipped three times.
   a. What is the probability that all three flips are heads?
   b. Given that the three flips are all heads, what is the probability that the biased coin was chosen?

4. Let $P = (X, Y)$ be a point chosen uniformly at random in the unit square $[0, 1]^2 = \{(x, y) : 0 \leq x, y \leq 1\}$. Find the cumulative distribution function for the random variable $Z = \min\{X, Y\}$, and then find its probability density function.