A transition probability matrix $P$ is regular if there is some $n > 0$ such that $P^n$ has all positive entries. On page 167 of An Introduction to Stochastic Modeling, Pinsky and Karlin claim that if an $N \times N$ transition matrix $P$ is regular, $P^{N^2}$ has all positive entries.

a. Find a regular $N \times N$ transition matrix $P$ such that $P^N$ has some 0 entries.

b. Find an $N \times N$ transition matrix $P$ such that $P, P^2, \ldots, P^n$ all have different patterns of positive entries, with $n > N^2$.

c. Does the existence of your example in (b) disprove Pinsky and Karlin’s claim?

d. If your answer to (c) is “no”, prove their claim.