Please simplify your answers to the extent reasonable without a calculator. Show your work. Explain your answers, concisely.

1. Two bugs start at opposite corners of a square. During each minute they each walk along an edge, chosen uniformly at random, to an adjacent corner. If they run into each other on an edge they stop, and if they end at the same corner, they stop. Let $T$ be the time at which they stop.

   a. [15 points] Describe this situation as a discrete time Markov chain: specify a set of states for the system and write down the one step probability transition matrix.
   
   b. [10 points] What is $E[T]$?
   
   c. [10 points] What would $E[T]$ be if the bugs start on adjacent vertices?

2. [30 points] Suppose that in California voters register each year as members of the Republican, Democratic, or Green parties, and that they change their registration randomly each year with transition probability matrix

   \[
   \begin{pmatrix}
   R & D & G \\
   R & 0.85 & 0.15 & 0 \\
   D & 0.05 & 0.85 & 0.10 \\
   G & 0 & 0.05 & 0.95
   \end{pmatrix}.
   \]

   In the long run, what fraction of voters will belong to each party?

3. Suppose when people get the flu and don’t stay home in bed they infect 2 other people with probability $p > 1/2$ and infect no one with probability $1 - p$.

   a. [15 points] What is the probability, starting with a single person with the flu, that the flu epidemic will die out?

   b. [20 points] If we could change people’s behavior so that when they get sick, they stay home in bed with probability $q > 0$. How big must $q$ be (in terms of $p$) to make the extinction probability for the flu be 1?

4. [Extra credit: 25 points] Consider the Markov chain describing aging with reincarnation: it has states $X_n \in \mathbb{N}$; at state $i \in \mathbb{N}$ there is probability $p_i$ of living one more year, to state $i + 1$, and probability $1 - p_i$ of dying and returning to state 0. Describe conditions on the $p_i$ that ensure that the state 0 is recurrent.