Not Sudoku

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We are going to make up our own multiplication tables for letters.* For example, instead of using 1, we will write e for the quantity that doesn't change something when it multiplies it.† The rules are:

- For every g, e times g (which we write as eg) is g. Also, $ge = g$.
- For every g, there is some h so that $gh = e$ and also $hg = e$.
  (h might be the same as g.)
- So inside our tables, in each row no letter appears twice.
  Also, in each column no letter appears twice.
- Finally, multiplication is *associative*: $(fg)h = f(gh)$.

1. Fill in the blank space in this multiplication table, following the rules.
   What is a times a: $aa = ?$

x	e	a
e	e	a
a		e

2. Fill in the blank spaces in this multiplication table, following the rules.
   What is aa?
   What is ab?
   What is ba?
   What is $abba$?

x	e	a	b
e	e	a	b
a	a		
b	b		

* These are *Cayley tables* for *finite groups*.

† e comes from "*Einheit*", which means "unit" in German.

3. Fill in the blank spaces in each of these 4 letter multiplication tables, following the rules:

x	e	a	b	c
e	e			
a		e		
b				e
c			e	

x	e	a	b	c
e	e			
a		e		
b			e	
c				e

4. Fill in the blank spaces in this 6 letter multiplication table, following the rules:

x	e	a	b	c	d	f
e	e					
a		e				
b			e			
c				e		
d						e
f					e	

What is ab?
What is ba?
What is $abba$?
What is $cafe$?