SDFAS
SAN DIEGO FRENCH ★ AMERICAN SCHOOL

Binomial coefficients (*Les coefficients binomiaux*)

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1. Suppose there is a platter with a kumquat, a persimmon, and a kiwi. How many ways are there to choose 1 of the 3 fruits? How many ways are there to choose 3 of the 3 fruits?

2. Suppose there are 4 slices of pizza left, and two people each take one slice. How many ways are there to choose the 2 slices remaining?

3. There is a special symbol for *the number of ways to choose k things out of n things*: $\binom{n}{k}$. (It is called a *binomial coefficient*.) Write the answers you found for the questions above:

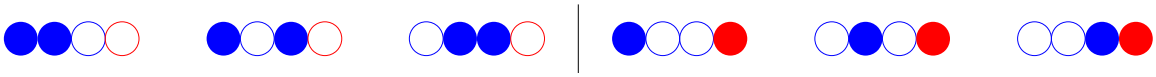
$$\binom{3}{1} = \qquad \binom{3}{3} = \qquad \binom{4}{2} =$$

4. Notice that in Problem 1, if you choose 1 fruit, there are 2 fruits left. So what is $\binom{3}{2}$?

5. Soon we'll learn how to calculate that $\binom{8}{3} = 56$, which means that there are 56 ways to choose 3 things out of 8. Each time you do that, there are 5 things remaining. So what is $\binom{8}{5}$?

6. The idea in the last two problems shows that $\binom{n}{k} = \binom{n}{n-k}$ because if we choose k things out of n, there are $n-k$ things remaining. What is $\binom{n}{n}$? So what is $\binom{n}{0}$?

7. What three "choosings" does this picture illustrate?



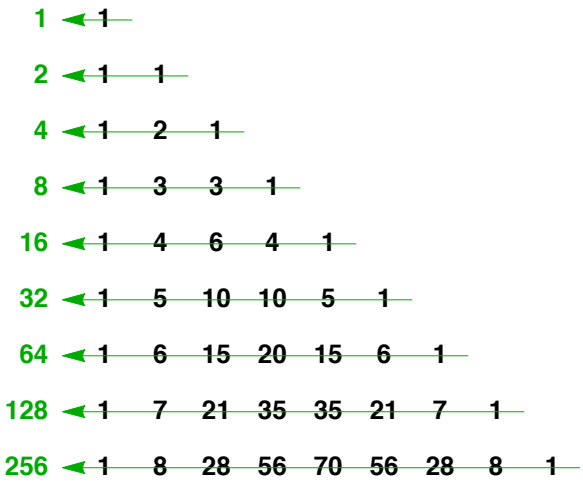
8. The idea in Problem 7 shows that $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$. Use this formula to copy the triangle below, replacing each binomial coefficient with a single number. (Remember what you discovered in Problem 6.)

$$
\begin{array}{ccccccccc}
\binom{0}{0} & & & & & & & & \\
\binom{1}{0} & \binom{1}{1} & & & & & & & \\
\binom{2}{0} & \binom{2}{1} & \binom{2}{2} & & & & & & \\
\binom{3}{0} & \binom{3}{1} & \binom{3}{2} & \binom{3}{3} & & & & & \\
\binom{4}{0} & \binom{4}{1} & \binom{4}{2} & \binom{4}{3} & \binom{4}{4} & & & & \\
\binom{5}{0} & \binom{5}{1} & \binom{5}{2} & \binom{5}{3} & \binom{5}{4} & \binom{5}{5} & & & \\
\binom{6}{0} & \binom{6}{1} & \binom{6}{2} & \binom{6}{3} & \binom{6}{4} & \binom{6}{5} & \binom{6}{6} & & \\
\binom{7}{0} & \binom{7}{1} & \binom{7}{2} & \binom{7}{3} & \binom{7}{4} & \binom{7}{5} & \binom{7}{6} & \binom{7}{7} & \\
\binom{8}{0} & \binom{8}{1} & \binom{8}{2} & \binom{8}{3} & \binom{8}{4} & \binom{8}{5} & \binom{8}{6} & \binom{8}{7} & \binom{8}{8}
\end{array}
$$

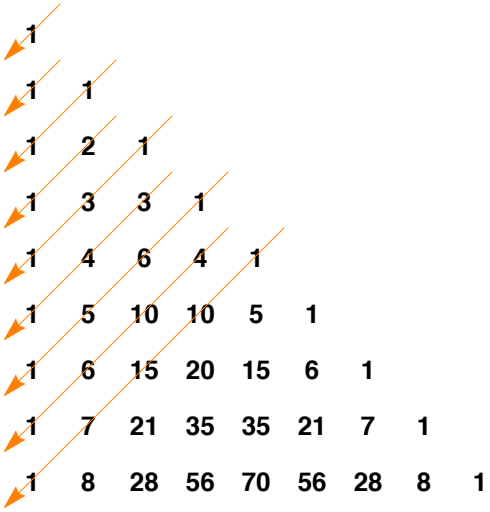
9. From the 3 fruits in Problem 1, we could choose the kumquat, or not; we could choose the persimmon, or not; and we could choose the kiwi, or not. How many different choices of fruits is this?

10. Find the sum of the numbers in each row of the triangle in Problem 8 (which is called *Pascal's Triangle*). Can you explain the pattern, based on the way you found the numbers in the triangle? Or based your answer to Problem 9?

11. The sums of the numbers in each of the first 9 rows of Pascal's triangle are the green numbers to which the horizontal green arrows point:



Find the sums of the numbers along each orange arrow:



Do you recognize these numbers? Can you explain the pattern, based on the way you found the numbers in the triangle?