

# Sums and products

dmeyer@math.ucsd.edu

You've all learned the multiplication table, at least up to  $10 \times 10$ . You also have learned the addition table, but probably didn't ever draw it. Today we'll draw both.\*

1. Fill in this *addition* table  $\rightarrow$

What do you notice about it? Hint: Color all the same numbers (that you fill in) with the same color and different numbers with different colors.

How many different colors did you use?

How many different colors would you need for a  $10 \times 10$  addition table?

+	1	2	3	4	5
1					
2					
3					
4					
5					

2. Fill in this *multiplication* table  $\rightarrow$

How many different colors do you need to color the part you fill in, still coloring the same numbers the same color and different numbers different colors?

Estimate how many different colors you would need for a  $10 \times 10$  multiplication table.

$\times$	1	2	3	4	5
1					
2					
3					
4					
5					

3. What is the number of colors in the addition table in Problem 1 plus the number of colors in the multiplication table in Problem 2?

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\* Inspired by G. Shakan, "On higher energy decompositions and the sum-product phenomenon", arXiv:1803.04637 [math.NT].



The numbers labeling the rows and columns of an addition or multiplication table do not have to be  $\{1, 2, 3, \dots\}$ .

4. Fill in the three boxes that you can in this multiplication table  $\rightarrow$

What should the number after the 2 be in order to make the numbers on the third diagonal (the yellow squares) all the same?

What should the rest of the numbers be, to continue the pattern of the same number on each diagonal? What kind of sequence do they make?

How many different colors do you need to color this table, still coloring the same numbers the same color and different numbers different colors?

<b>×</b>	<b>1</b>	<b>2</b>			
<b>1</b>					
<b>2</b>					

5. Fill in this addition table using the same numbers labeling the rows and columns as in Problem 4  $\rightarrow$

How many different colors do you need to color this table, still coloring the same numbers the same color and different numbers different colors?

<b>+</b>	<b>1</b>	<b>2</b>			
<b>1</b>					
<b>2</b>					

6. Which is smaller, the number of colors in Problem 1 plus the number of colors in Problem 2, or the number of colors in Problem 4 plus the number of colors in Problem 5?

