Binary numbers

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We discovered that sea cucumbers only use two numbers, C• and ◖, which mean 0 and 1, respectively. So the way they count is:

$$\begin{aligned} 0 &= 0 \\ 1 &= 1 \\ 10 &= 2 \\ 11 &= 3 \\ 100 &= 4 \\ 101 &= 5 \\ 110 &= 6 \\ &\vdots \end{aligned}$$

The sea cucumber way of writing numbers is called the binary number system (*le système binaire*), and is the way electronic computers represent numbers internally. The number system you have learned in school is the decimal system (*le système décimal*).

1. How do you write 7 in binary? 32? 31?
   What is 1101 in decimal? 1100001?

2. Adding in binary is very easy:

$$\begin{array}{r} {\scriptstyle 1\ 1\ 1\ \ } \\ 1\,0\,1 \\ +\quad 1\,1 \\ \hline 1\,0\,0\,0 \end{array} \qquad \begin{array}{r} 5 \\ +\,3 \\ \hline 8 \end{array}$$

Try these addition problems:

$$\begin{array}{r} 1101 \\ +\quad 1 \\ \hline \end{array} \qquad \begin{array}{r} 10110 \\ +\ 1111 \\ \hline \end{array} \qquad \begin{array}{r} 1100001 \\ +\quad 11100 \\ \hline \end{array}$$

3. And these subtraction problems:

$$\begin{array}{r} 1101 \\ -\quad 1 \\ \hline \end{array} \qquad \begin{array}{r} 10110 \\ -\ 1111 \\ \hline \end{array} \qquad \begin{array}{r} 1100001 \\ -\quad 11100 \\ \hline \end{array}$$

4. Multiplication works the same way in binary as it does in decimal—try it:

$$\begin{array}{r} 101 \\ \times\ 11 \\ \hline \end{array} \qquad \begin{array}{r} 10110 \\ \times\ 10 \\ \hline \end{array} \qquad \begin{array}{r} 1100001 \\ \times\ 1100 \\ \hline \end{array}$$

5. Which of these binary numbers is evenly divisible by 2? 1101, 10110, 1100001, 11100. Are any of them evenly divisible by 4?

6. Remember that to check if a decimal number is evenly divisible by 3, we learned to add up its digits and check that sum is evenly divisible by 3. For example: 3625419 is divisible by 3 because $3 + 6 + 2 + 5 + 4 + 1 + 9 = 30$, which is evenly divisible by 3. Does this work for binary numbers? If not, can you find a different way to check? Try 10101, 101010, 11, 111, 11100001.

7. Checking if a decimal number is evenly divisible by 10 is very easy. Can you find a way to check if a binary number is evenly divisible by 10? Try your method on 11100001, 10100010.

8. How do you write 0.5 in binary? 0.25? 0.75?
   What is 0.001 in decimal? $0.0\overline{1}$?