Math 106 - Midterm 1.

The exam consists of 6 questions. Each page of the exam is worth 10 points. The maximum number of points is 50.

Name:

Acknowledgement and acceptance of honor code:

Signature:

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Problem 1 (10 points)

Write the following complex numbers in standard form $a + bi$:

(i) [5]

$$\exp \left( -1 + \frac{i\pi}{4} \right)$$

(ii) [5]

$$\frac{1 + 3i}{1 - 2i}$$
Problem 2 (10 points)

(i) [5] Determine the domain of definition of the function

\[ f(z) = \frac{z^2 + 1}{z^2 - 16i}. \]

(ii) [5] Compute the derivative of the function

\[ f(z) = \frac{z - 1}{2z + 1} \]

at \( z = -1 + \frac{i}{2} \).
Problem 3 (10 points)

True or false:

(i) [2] \( \sqrt{z} \) is an entire function.

(ii) [2] the complex numbers \( z \) with \( |z - 2 + 3i| = 1 \) form a line in the complex plane.

(iii) [2] a function \( f \) is holomorphic if and only if \( \frac{\partial f}{\partial \bar{z}} = 0 \).

(iv) [2] \( \lim_{z \to \infty} e^z = \infty \).

(v) [2] \( u(x, y) = x^3 + y^3 \) is harmonic.
**Problem 4 (10 points)**

Let \( u(x, y) = x - y + 2x^2 - 2y^2 \).

(i) [6] Check that \( u \) is harmonic. Find a holomorphic function \( f \) whose real part is \( u(x, y) \) and \( f(0) = 0 \).

(ii) [4] Write \( f \) as a function of \( z \) alone.
Problem 5 (5 points)

Using the Cauchy-Riemann equations verify that the function

\[ f(x, y) = 2x^2 - 2y^2 + 2xy + i(-x^2 + y^2 + 4xy) \]

is entire.

Problem 6 (5 points)

Determine the 9th derivative of the function

\[ f(z) = \exp \left( (-1 + i\sqrt{3})z \right) \]

at \( z = 0 \).