4 sides. Note $f = 2(x+iy) + (x^2-y^2+2ixy) = (2x+x^2-y^2) + i(2y+2xy) \Rightarrow$

$F = |f|^2 = (x^2-y^2+2x)^2 + 4(xy+y)^2$. We want max F.

- When $x=-1$, $F = (-y^2-1)^2 = \max \iff y^2 \max \iff y=\pm 1$. $\underline{F=4}$.
- When $x=1$, $F = (3-y^2)^2+16y^2 = y^4+10y^2+9$. is increasing in y^2 so max

when $y^2=1 \Rightarrow F=20$, $|f| = \sqrt{20}$, $\boxed{z = 1\pm i}$

- When $y=\pm 1$, $F = (x^2+2x-1)^2 + 4(x+1)^2$., $F_x = 2(2x+2)(x^2+2x-1)+8(x+1) =$

$= 4(x+1)(x^2+2x-1) + 8(x+1) = 4(x+1)(x^2+2x+1) = 8(x+1)^3 \geq 0$. Thus F is max $\underline{\text{again}}$

when $x=1 \Rightarrow \boxed{z = 1\pm i}$ (again) and then $|f| = \sqrt{20}$.

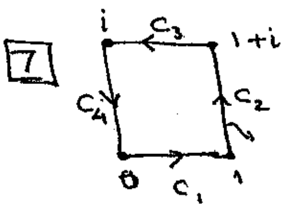
$\boxed{6}$. (i). The pole $z=-1$ is not in $\mathfrak{H}^+$ so c is holomorphic in $\mathfrak{H}^+$. If $z\in\mathfrak{H}^+$, write $z = x+iy, x>0$. We check $|c(z)|<1$ for $z\in\mathfrak{H}^+$. This means $\left|\frac{z-1}{z+1}\right|<1 \iff |z-1|^2<|z+1|^2$

$\iff (x-1)^2+y^2 < (x+1)^2+y^2 \iff 4x>0$ which is true.

(ii). If $f(z)\in\Delta$ for all z then $|f(z)|<1$. Since f is entire, Liouville's thm implies f is constant. The answer is then $\boxed{\text{no}}$

(iii). If $f(z)\in\mathfrak{H}^+$ for all z then $c(f(z))$ is well defined. and since $c:\mathfrak{H}^+\longrightarrow\Delta$ we have $c(f(z))$ is a holomorphic function with values in Δ so it must be a constant c. Then $c = c(f(z)) = \frac{f(z)-1}{f(z)+1} \iff f(z) = -\frac{1+c}{1-c}$ also constant!! The answer is again $\boxed{\text{no}}$

$\boxed{7}$



$$\int_{C_1}\pi e^{\pi\bar z}\,dz = \int_0^1 \pi e^{\pi t}\,dt = e^{\pi t}\Big|_0^1 = e^{\pi}-1.$$

$$\int_{C_2}\pi e^{\pi\bar z}\,dz = \int_0^1 \pi e^{\pi(1-it)}\,d(1+it) = -e^{\pi(1-it)}\Big|_0^1 =$$

$$= -e^{\pi(1-i)} + e^{\pi} = e^{\pi}+e^{\pi} = 2e^{\pi}.$$

$\underline{\text{On } C_3}$, $z = i+(1-t)$, $0\le t\le 1$. $\int_{C_3}\pi e^{\pi\bar z}\,dz = \int_{C_3}\pi e^{\pi(1-t-i)}(-dt) = \cancel{\pi}\, e^{\pi(1-t-i)}\Big|_0^1 =$

$= e^{-\pi i} - e^{\pi(1-i)} = e^{\pi}-1.$

$\underline{\text{On } C_4}$: $z = i(1-t)$, $0\le t\le 1$. $\int_{C_4}\pi e^{\pi\bar z}\,dz = \int_0^1 \pi e^{+i(t-1)\pi}\, i(-dt) = -e^{i(t-1)\pi}\Big|_0^1 = -e^0 + e^{-\pi} =$

$= -2.$

$\underline{\text{Answer}}$ $\boxed{\int_C \pi e^{\pi\bar z}\,dz = 4(e^{\pi}-1)}$