(i) \[ \text{Res } \frac{z - \sin z}{z^2} = \frac{\pi}{6} \]

Answer: \( \frac{\pi}{6} \)

(ii) \[ \text{Res } \frac{\sqrt{z}}{z^2 + 1} = \text{Res } \frac{f(z)}{1 + z^2} = \frac{f(-i)}{2} \]

where \( f(z) = \frac{z - \sin z}{z^2} \)

Answer: \( \frac{\sqrt{i}}{\sqrt{2}} \)

(iii) \( \frac{f(-i)}{1 + z^2} = \frac{1}{\sqrt{2}} \left( \frac{1 + i}{1 - i} \right) \)

Answer: \( \frac{\sqrt{i}}{\sqrt{2}} \)

5. \( f(z) = \frac{1}{z+1} - \frac{1}{z+4} \)

When \( |z| < 1 \), \[ \frac{1}{1+z} = 1 - 2 + 2^2 - 2^3 + \ldots \]

When \( |z| > 1 \), \[ \frac{1}{z+1} = \frac{1}{z} \cdot \frac{1}{1 + \frac{1}{z}} = \frac{1}{z} \left( 1 - \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots \right) \]

When \( 1 < |z| < 4 \), \[ f(z) = \left( 1 - \frac{1}{4} \right) z \left( 1 + \frac{1}{4^2} \right) + 2^2 \left( 1 - \frac{1}{4^3} \right) - \ldots \]

When \( |z| > 4 \), \[ f(z) = \frac{3}{z^2} + \frac{1 - 4^2}{2^3} + \frac{-1 + 4^3}{2^4} + \ldots \]

10. Isolated singularities at 0, \( \alpha \), \( \beta \), all are poles.

\[ \text{Res } f(z) = \frac{1}{a \ b} \]

\[ \text{Res } f(z) = \frac{1}{z-a} \]

\[ \text{Res } f(z) = \frac{1}{z-a} \]

\[ \text{Res } f(z) = \frac{1}{z-b} \]

\[ \text{Res } f(z) = \frac{1}{z-b} \]

\[ \int_C f(z) \, dz = 2\pi i \left( \frac{1}{ab} + \frac{1}{a-b} + \frac{1}{b} \right) = 2\pi i \left( \frac{a-b}{ab} \right) \]

\[ \frac{1}{a-b} + \frac{1}{a-b} = \frac{a+b}{ab} \]

\[ \frac{1}{z-a} = \frac{1}{\frac{a}{2}} \left( 1 + \frac{a}{2} + \ldots \right) \]

\[ \frac{1}{z-b} = \frac{1}{\frac{b}{2}} \left( 1 + \frac{b}{2} + \ldots \right) \]

\[ \frac{f(z)}{2^3} \left( \frac{1}{a-b} + \frac{1}{a-b} \right) = \frac{1}{2^3} \left( \frac{a}{b} + \frac{b}{a} \right) \left( \frac{1}{2} + \frac{1}{2} + \ldots \right) \]

\[ \frac{f(z)}{2^3} \left( \frac{1}{a-b} + \frac{1}{a-b} \right) = \frac{1}{2^3} \left( \frac{a}{b} + \frac{b}{a} \right) \left( \frac{1}{2} + \frac{1}{2} + \ldots \right) \]