There's no $\frac{1}{z}$, but this doesn't mean that the residue of f at 0 is 0 <u>because</u> this expansion is valid too far away from 0.

(iv). All terms appearing in $(*)$: $\frac{1}{z^3}, \frac{a+b}{z^4}, \frac{a^2+ab+b^2}{z^5}\ldots$ integrate to 0 on any closed contour by F.T.C. (they have anti-derivatives) $\Longrightarrow \int_C f = \boxed{0}$.

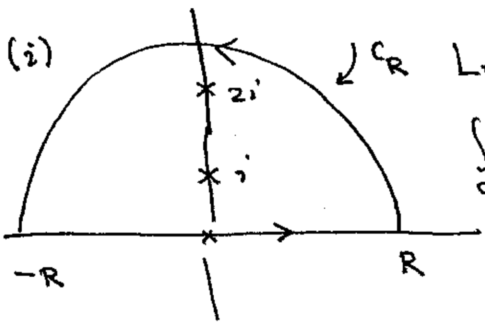
(v). If C only encloses the poles $z=0$, $z=a$, the answer becomes

$$2\pi i\left(\frac{1}{ab}+\frac{1}{a(a-b)}\right) = \boxed{2\pi i \,\frac{1}{b(a-b)}} \neq 0.$$

(vi). By the same argument $\frac{1}{z-a_i} = \frac{1}{z}+\frac{a_i}{z^2}+\cdots \Rightarrow \frac{1}{(z-a_1)\cdots(z-a_n)} =$ valid for $|z|$ large

$= \frac{1}{z^n}+\frac{a_1+\dots+a_n}{z^{n+1}}+\dots$ If P has degree at most $n-2$, $\frac{P(z)}{(z-a_1)\cdots(z-a_n)}$ has no powers of $\frac{1}{z}$ in its expansion so $\int_C \frac{P(z)\,dz}{(z-a_1)\cdots(z-a_n)} = 0$.

$\boxed{11}$ (i)



Let $C = C_R$ + the interval from $-R$ to R, with $R>2$.

$$\int_C \frac{z^2\,dz}{(z^2+1)(z^2+4)} = 2\pi i\left(\operatorname*{Res}_{z=i} + \operatorname*{Res}_{z=2i}\right)$$

$$\operatorname*{Res}_{z=i} \frac{z^2}{(z^2+1)(z^2+4)} = \frac{f(i)}{g'(i)} = \frac{i^2}{i^2+4}\cdot\frac{1}{2i} \text{ with } f=\frac{z^2}{z^2+1}, g=z^2$$

$$= -\frac{1}{6i}$$

$$\operatorname*{Res}_{z=2i} \frac{z^2}{(z^2+1)(z^2+4)} = \frac{(2i)^2}{(2i)^2+1}\cdot\frac{1}{2(2i)} = \frac{-4}{1-4}\cdot\frac{1}{4i} = \frac{1}{3i}.$$

$$\Longrightarrow \int_C \frac{z^2\,dz}{(z^2+1)(z^2+4)} = 2\pi i\left(-\frac{1}{6i}+\frac{1}{3i}\right) = \boxed{\frac{\pi}{3}}$$

<u>Now</u> $\int_C = \int_{C_R} + 2\int_0^R \frac{x^2\,dx}{(x^2+1)(x^2+4)}$.

$$\left|\int_{C_R} \frac{z^2\,dz}{(z^2+1)(z^2+4)}\right| \le \int_{C_R} \frac{R^2}{(R^2-1)(R^2-4)}\,ds = \frac{R^2}{(R^2-1)(R^2-4)}\cdot \pi R \longrightarrow 0 \text{ as } R\to\infty. \text{ Thus}$$

$$\boxed{\int_0^\infty \frac{x^2}{(x^2+1)(x^2+4)}\,dx = \frac{\pi}{6}}$$

(ii). Use the same contour C as above for $\int_C \frac{e^{iaz}}{(z^2+b^2)^2}\,dz$. The <u>residue</u> at bi

$$\operatorname*{Res}_{z=bi} \frac{e^{iaz}}{(z^2+b^2)^2} = \operatorname*{Res}_{z=bi} \frac{F(z)}{(z-bi)^2} = F'(bi) \text{ with } F(z) = \frac{e^{aiz}}{(z+bi)^2},\ F'(z) =$$