

$$= \frac{e^{aiz} (a^2) (z+bi)^2 - e^{aiz} \cdot 2(z+bi)}{(z+bi)^4} \Rightarrow F'(bi) = \frac{e^{-ab} \cdot (a^2) (-4b^3) - e^{-ab} \cdot 2 \cdot (2bi)}{(2bi)^4} =$$

$$= \frac{-e^{-ab} i (ab+1)}{4b^3} \Rightarrow \int_C \frac{e^{iaz}}{(z^2+b^2)^2} dz = 2\pi i \cdot \text{Res} = \boxed{\frac{\pi \cdot e^{-ab} (ab+1)}{2b^3}}$$

$$\left| \int_{C_R} \frac{e^{aiz}}{(z^2+b^2)^2} dz \right| \leq \int_{C_R} \frac{|e^{aiz}|}{(R^2-b^2)^2} ds = \frac{\pi R}{(R^2-b^2)^2} \cdot \underbrace{e^{-a \cdot \text{Im}z}}_{\leq 1} \rightarrow 0 \text{ as } R \rightarrow \infty$$

$$\int_C = \int_{-R}^R \frac{e^{aiz}}{(z^2+b^2)^2} dz + \int_{C_R} \frac{e^{aiz}}{(z^2+b^2)^2} dz = \frac{\pi}{2b^3} e^{-ab} (ab+1) \text{ take real part}$$

$$\Rightarrow \lim_{R \rightarrow \infty} \int_{-R}^R \frac{\cos az}{(z^2+b^2)^2} dz = \frac{\pi}{2b^3} e^{-ab} (ab+1) \Rightarrow \boxed{\int_0^{\infty} \frac{\cos(az)}{(z^2+b^2)^2} dz = \frac{\pi}{4b^3} e^{-ab} (ab+1)}$$

12 Let $f(z) = 3z^3$, $g(z) = 3z^3 - e^z$. We claim that $|f| > |f-g|$ on the boundary of R
 $\Leftrightarrow 3|z|^3 > |e^z|$ on the boundary of R . If z is on the boundary, $\text{Re } z \leq 1$, $|z| \geq 1$.

$\Rightarrow 3|z|^3 \geq 3 \geq e \geq e^{\text{Re } z} = |e^z|$ proving the claim. Thus, by Rouché's, g must have 3 zeroes inside R , because f has 3 zeroes inside R (counted with multiplicities).

Assume a is not a simple zero of g . $\Rightarrow g(a) = g'(a) = 0 \Leftrightarrow 3a^3 - e^a = 3a^2 - e^a = 0$
 $\Leftrightarrow 3a^3 = 3a^2 = e^a \Leftrightarrow a = 0$ or 3 ; but $g(0), g(3) \neq 0$ contradiction! The three zeroes of g must therefore be simple.