Math 10C - Fall 2017 - Final Exam

Name: ________________________________

Student ID: __________________________

Section time: _________________________

Instructions:

Please print your name, student ID and section time.

During the test, you may not use books, calculators or telephones. You may use a "cheat sheet" of notes which should be a page, front only.

Read each question carefully, and show all your work. Answers with no explanation will receive no credit, even if they are correct.

There are 9 questions which are worth 105 points. You have 180 minutes to complete the test.

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Problem 1. [12 points; 4, 4, 4.]
Consider the function

\[ f(x, y) = 1 - x^2 - (y - 1)^2. \]

(i) Draw the level curve through the point \( P(1, 2) \). Find the gradient of \( f \) at the point \( P \) and draw the gradient vector on the level curve.

(ii) Draw the graph of \( f \) showing the level curve in (i) on the graph.
(iii) Explain why the function $f$ admits a global minimum over the rectangle

$$0 \leq x \leq 2, \ -1 \leq y \leq 1.$$ 

Determine the minimum value and the point(s) where it occurs.
Problem 2. [12 points.]

Consider the integral
\[ \int_0^4 \int_{\sqrt{y}}^{2} ye^{x^5} \, dx \, dy. \]

Draw the region of integration and then evaluate the integral by changing the order of integration.
Problem 3. [12 points; 5, 7.]

(i) Find the critical points of the function

\[ f(x, y) = x^3 + 3xy - y^3. \]
(ii) Indicate the type of the critical points.
Problem 4. [12 points.]

Find the global minimum and global maximum of the function

\[ f(x, y) = x^2 + 2y^2 - 2x - 8y + 9 \]

over the region

\[ x^2 + 2y^2 \leq 36. \]
Problem 5. [10 points.]

Assume that
\[ z = \sqrt{ye^x}, \quad x = u - uv, \quad y = \frac{u}{v}, \quad u = 3s + t, \quad v = t^2. \]

Compute \( \frac{\partial z}{\partial s} \) at the point where \( s = t = 1 \).
Problem 6. [10 points; 5, 5.]

Consider the points \( P(1, 0, 1), \ Q(-2, 1, 3), \ R(1, -1, 0). \)

(i) Find the equation of the plane through \( P, Q, R. \)

(ii) Find the cosine of the angle between the vectors \( \vec{PQ} \) and \( \vec{PR}. \)
Problem 7. [13 points; 4, 4, 5.]

Consider the function
\[ f(x, y) = 2y \ln(x - \sqrt{y}). \]

(i) Find the unit direction of steepest increase for \( f \) at the point \( P(2, 1) \).

(ii) Find the directional derivative of \( f \) at the point \( P(2, 1) \) in the direction \( \vec{u} = \frac{3\vec{i} - 4\vec{j}}{5} \).

(iii) Linearly approximate the value \( f((2, 1) + \frac{1}{200} \vec{u}) \).
**Problem 8.** [12 points; 5, 7.]

Consider the function $f(x, y) = x^2 y^3 - e^{xy} - 1$.

(i) Find the tangent plane to the graph of $f$ at $(1, 1, 0)$. 

(ii) Find the quadratic approximation of the function $f$ near $(1,1)$. 
**Problem 9.** [12 points.]

Consider the triangular $T$ region bounded by

\[ x \geq 0, \ y \geq 0, \ x + 2y \leq 2. \]

Calculate

\[ \int \int_T x + y \ dx \ dy. \]