Math 10C - Fall 2009 - Midterm II

Name: ________________________________

Student ID: __________________________

Section time: ________________________

Instructions:

Please print your name, student ID and section time.

During the test, you may not use books or telephones. You may use a "cheat sheet" of notes which should be a page, front only.

Read each question carefully, and show all your work. Answers with no explanation will receive no credit, even if they are correct.

There are 5 questions which are worth 50 points. You have 50 minutes to complete the test.

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Problem 1. [10 points.]

Consider the points $P, Q, R$ with coordinates $(2, 1, 0), (0, 1, 3)$ and $(1, 0, 1)$ respectively.

(i) Find the area of the parallelogram spanned by the vectors $\vec{PQ}$ and $\vec{PR}$.
(ii) Find the equation of the plane through $P, Q, R$. 
Problem 2. [10 points.]

Find the second order Taylor polynomial near (1, −1) for the function

\[ f(x, y) = x^3 y. \]
Problem 3. [10 points.]

Consider the function

\[ f(x, y) = x^4y^3. \]

(i) [4] Write down the equation of the tangent plane at the graph of the function at the point \((1, 1, 1)\).

(ii) [4] Write down an expression for the change, \(\Delta z\), in \(z = f(x, y)\) depending on \(\Delta x\) and \(\Delta y\), the change in \(x\) and \(y\), respectively, near the point \(x = y = 1\). Is the function \(f(x, y)\) more sensitive to a change in \(x\) or to a change in \(y\)?
(iii) [3] Using your answer to (ii), find the approximate value of $f(1.01, 1.01)$.

Problem 4. [10 points]
Consider the function $f(x, y) = xe^{x+y}$ and the point $P = (2, -22)$.

(i) [4] Find the gradient of $f$ at $P$. 
(ii) [3] Find the directional derivative of $f$ at $P$ in the direction $\mathbf{u} = \frac{1}{\sqrt{2}}(\mathbf{i} - \mathbf{j})$.

(iii) [3] What is the direction of steepest increase for the function $f$ at $P$? Express your answer as a unit vector.
Problem 5. \([10 \text{ points}]\)
Consider the function
\[
w = \sin(xy)
\]
where
\[
x = \frac{1}{v}, \quad y = u^2v.
\]
Using the chain rule, calculate the derivatives
\[
\frac{\partial w}{\partial u} \quad \text{and} \quad \frac{\partial w}{\partial v}.
\]
Please express your answer in simplest form.