Instructions:

Please print your name, student ID and section time.

During the test, you may not use books or telephones. You may use a "cheat sheet" of notes which should be a page, front only.

Read each question carefully, and show all your work. Answers with no explanation will receive no credit, even if they are correct.

There are 4 questions which are worth 40 points. You have 50 minutes to complete the test.

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Problem 1. [10 points; 3, 3, 4.]

Consider the function
\[ f(x, y) = e^{-y} x^3. \]

(i) Find the equation of the tangent plane to the graph \( z = f(x, y) \) at \( (1, 0, 1) \).

(ii) Using linear approximation, estimate \( f(1.01, .01) \).
(iii) Calculate the quadratic Taylor polynomial of $f$ near $(1, 0)$. 
Problem 2. [10 points; 4, 3, 3.]

Consider the function

\[ f(x, y) = x^2 \sin(y - 2x) \]

and the point \( P(1, \pi + 2) \).

(i) Find the gradient of \( f \) at the point \( P \).

(ii) Calculate the directional derivative of \( f \) at \( P \) in the direction \( \vec{u} = 3\vec{i} + 4\vec{j} \).

\[ \vec{u} = \frac{3\vec{i} + 4\vec{j}}{5}. \]
(iii) Find the (unit) direction of steepest increase for the function $f(x, y)$ at $P$. 
**Problem 3.** [10 points.]

Find the critical points of the function

\[ f(x, y) = 2x^3 - 3x^2y - 12x^2 - 3y^2 \]

and determine their type i.e. local min/local max/saddle point. Are there any global min/max?
Problem 4. [10 points.]

Consider the function

$$z = y \ln x$$

and assume

$$x = e^{2u}, \quad y = \frac{v^2}{u}.$$

Using the chain rule, calculate the derivatives

$$\frac{\partial z}{\partial u} \text{ and } \frac{\partial z}{\partial v}.$$

Please express your answer in the simplest form.