**Problem 1.**

Consider the function 

\[ f(x, y) = x^2 + y^2 + 2. \]

(i) Draw the contour diagram for \( f(x, y) \) and clearly label the level curves. Show the contours for at least three levels.

*The level curve at level \( c \) is*

\[ f(x, y) = c \implies x^2 + y^2 + 2 = c \implies x^2 + y^2 = c - 2. \]

*This is a circle with center at the origin and radius \( \sqrt{c - 2} \).*

For instance you may pick the following three levels:

- \( c = 3 \) which gives a circle of radius 1 with center at the origin. This circle is labeled by the level 3;
- \( c = 4 \) which gives a circle of radius \( \sqrt{2} \) labeled by the level value 4;
- \( c = 5 \) which gives a circle of radius \( \sqrt{3} \) labeled by the level value 5;

There are other choices for levels, so other answers may be correct as well.

(ii) Draw the graph of \( z = f(x, y) \).

*The graph is a paraboloid whose lowest point is \((0, 0, 2)\).*
**Problem 2.**

(i) Carefully draw the plane \( z = 6 - 3x - 2y \) and calculate the points where the plane intercepts the coordinate axes.

*The plane cuts the x axis for y = z = 0 so this means 0 = z = 6 - 3x - 2 \cdot 0 \text{ so } x = 2.*

Similarly the plane cuts the y-axis by setting \( x = z = 0 \) which yields \( 6 - 2y = 0 \text{ so } y = 3. *Finally, the plane cuts the z axis for x = y = 0 so z = 6. Thus the plane intersects the coordinate axes at (2,0,0), (0,3,0) and (0,0,6).*

The plane can be drawn by joining the points (2,0,0), (0,3,0) and (0,0,6).

(ii) Find the equation of the plane that passes through the points (1,1,0), (3,1,−2) and (3,0,1).

*We consider the points (1,1,0) and (3,1,−2). Their second coordinates agree, so we can use these points to compute the m-slope:*

\[
m = \frac{-2 - 0}{3 - 1} = -1.
\]

*Similarly, we can use the points (3,1,−2) and (3,0,1) to compute the n-slope:*

\[
n = \frac{1 - (-2)}{0 - 1} = -3.
\]

*Finally, we can write down the equation of the plane when we know the slopes m and n and one of the points (1,1,0):*

\[
z = -(x - 1) - 3(y - 1) \implies z = -x - 3y + 4.
\]

*The same problem can be solved using cross products.*
Problem 3.

Consider the vectors
\[ \vec{v} = \vec{i} + \vec{j} - \vec{k}, \quad \vec{w} = 2\vec{i} - \vec{j} + \vec{k}. \]

(i) Compute \( \vec{v} \cdot \vec{w} \) and find the angle between the two vectors.

Direct computation shows
\[ \vec{v} \cdot \vec{w} = (1, 1, -1) \cdot (2, -1, 1) = 1 \cdot 2 + 1 \cdot (-1) + (-1) \cdot 1 = 0. \]

Thus the vectors \( \vec{v}, \vec{w} \) are perpendicular. The angle between them is 90 degrees.

(ii) Find a unit vector perpendicular to both \( \vec{v} \) and \( \vec{w} \).

We compute first the cross product
\[ \vec{v} \times \vec{w} = \vec{i}(1 - 1) - \vec{j}(1 - (-2)) + \vec{k}(-1 - 2) = -3\vec{j} - 3\vec{k}. \]

The cross product is perpendicular to both \( \vec{v} \) and \( \vec{w} \). The magnitude of this vector equals
\[ ||\vec{v} \times \vec{w}|| = \sqrt{(-3)^2 + (-3)^2} = \sqrt{18} = 3\sqrt{2}. \]

The unit vector is
\[ \vec{u} = \frac{\vec{v} \times \vec{w}}{||\vec{v} \times \vec{w}||} = \frac{-3\vec{j} - 3\vec{k}}{3\sqrt{2}} = \frac{-1}{\sqrt{2}} \vec{j} - \frac{1}{\sqrt{2}} \vec{k}. \]

(iii) Find the area of the parallelogram spanned by \( \vec{v} \) and \( \vec{w} \).

The area of the parallelogram is the magnitude of the cross product
\[ ||\vec{v} \times \vec{w}|| = 3\sqrt{2} \]
as computed above.
Problem 4.

(i) Find the normal vectors to the planes
\[ x - y + 2z = 2, \quad 3x - y + 2z = 1. \]

We read off the normal vectors from the equations of the planes. We have
\[ \vec{n}_1 = (1, -1, 2), \quad \vec{n}_2 = (3, -1, 2). \]

(ii) Find a vector parallel to the line of intersection of the planes
\[ x - y + 2z = 2, \quad 3x - y + 2z = 1. \]

The line of intersection will be perpendicular to both \( \vec{n}_1, \vec{n}_2 \). But so is the cross product. Thus the line of intersection will be parallel to the cross product
\[ \vec{n}_1 \times \vec{n}_2 = (1, -1, 2) \times (3, -1, 2) = (0, 4, 2). \]
We obtain the vector \( 4\vec{j} + 2\vec{k} \).

(iii) Find the plane through the origin parallel to
\[ z = 4x - 3y + 8. \]

We rewrite the plane as
\[ 4x - 3y - z = -8. \]

The normal vector for the plane is \( \vec{n} = (4, -3, -1) \). The second plane parallel to it must have the same normal vector, hence the same coefficients for \( x, y, z \). Since it passes through the origin, the equation is
\[ 4x - 3y - z = 0. \]