Math 140B - Spring 2017 - Midterm II

Name: _______________________________________

Student ID: _______________________________

Instructions:

Please print your name, student ID.

During the test, you may not use books, notes or telephones.

Read each question carefully, and show all your work. Answers with no explanation will receive no credit, even if they are correct.

There are 4 questions which are worth 40 points total. You have 50 minutes to complete the test.

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Problem 1. [10 points; 4, 6.]

Let $f_n : (0, 1) \to \mathbb{R}$ be given by

$$f_n(x) = \frac{n}{1 + nx}.$$

(i) Find the pointwise the limit $f$ of the sequence $f_n$ over the interval $(0, 1)$.

(ii) Let $a > 0$. Calculate $M_n = \sup_{x \in [a, 1)} |f_n(x) - f(x)|$ and show that the sequence $f_n$ converges uniformly to $f$ over the interval $[a, 1)$.
Problem 2. [10 points; 5, 5.]

Consider the series

\[ f(x) = \sum_{n=1}^{\infty} \frac{1}{n^2 x^2 + 1}. \]

(i) Show that \( f \) converges uniformly over \([1, \infty)\).

(ii) Show that \( f \) is differentiable over \([1, \infty)\).
Problem 3. [10 points.]

Let \( f_n : \mathbb{R} \rightarrow \mathbb{R} \) be a sequence of functions which is uniformly bounded and equicontinuous. Show that the sequence \( f_n^2 : \mathbb{R} \rightarrow \mathbb{R} \) is also equicontinuous.
Problem 4. [10 points.]

Assume the functions $f_n : \mathbb{R} \rightarrow \mathbb{R}$ converge uniformly to $f : \mathbb{R} \rightarrow \mathbb{R}$. Show that if $f_n$ are uniformly continuous then $f$ is uniformly continuous.