Name: ____________________________

Student ID: ________________________

Section time: ______________________

Instructions:

Please print your name, student ID and section time.

During the test, you may not use books, calculators or telephones. You may use a "cheat sheet" of notes which should be at most half a page, front and back.

Read each question carefully, and show all your work. Answers with no explanation will receive no credit, even if they are correct.

There are 5 questions which are worth 45 points. You have 50 minutes to complete the test.

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Problem 1. [7 points.]

Find the volume of the region bounded above by the elliptic paraboloid \( z = 4 - x^2 - 3y^2 \), on the bottom by the \((x, y)\)-plane, on the sides by the planes \( x = 0 \), \( x = 1 \), \( y = -1 \) and \( y = 1 \).
Problem 2. [9 points.]

Find the critical points of the function

\[ f(x, y) = x^3 + 6xy + 3y^2 - 9x \]

and determine their nature.
Problem 3. [8 points.]

Let \( z = f(x, y) \) where \( f \) is a function such that
\[
\frac{\partial f}{\partial y} = x - 2y.
\]
It is furthermore known that
\[
\frac{\partial f}{\partial x}(4, 3) = 2.
\]
Assume that
\[
x = \frac{u^2}{v}, \quad y = 2uv - v^2.
\]
Calculate the derivative
\[
\frac{\partial z}{\partial v}
\]
at the point \( u = 2 \) and \( v = 1 \).
Problem 4. [9 points.]

Find the minimum and the maximum value of the function

\[ f(x, y) = x^2 + 2y^2 - 6x + 2 \]

along the ellipse

\[ 2x^2 + y^2 = 8. \]
Problem 5. [12 points; 3, 2, 3, 4.]

Consider the function
\[ f(x, y) = x^2 y^4 + xy^2 \ln(2x - y). \]

(i) Find the direction of steepest increase for the function \( f \) at the point \((1,1)\).

(ii) Find the directional derivative \( D_{\vec{v}} f(1,1) \) in the direction \( \vec{v} = \frac{1}{\sqrt{2}} \vec{i} - \frac{1}{\sqrt{2}} \vec{j} \).
(iii) Find the tangent plane to the graph of \( f \) at the point \((1, 1, 1)\).

(iv) Find the tangent plane to the level surface \( z^2x^3 - f(x, y) = 0 \) at the point \((1, 1, 1)\).