Math 20 C - Fall 2011 - Final Exam

Name: ______________________________

Student ID: ______________________________

Section time: ______________________________

Instructions:

Please print your name, student ID and section time.

During the test, you may not use books, calculators or telephones. You may use a "cheat sheet" of notes which should be at most one page.

Read each question carefully, and show all your work. Answers with no explanation will receive no credit, even if they are correct. You have 3 hours to complete the test.

Question	Score	Maximum
1		8
2		10
3		7
4		13
5		10
6		13
7		13
8		13
9		10
10		12
11		13
12		8
Total		130

Problem 1. [*8 points.*]

Evaluate by changing the order of integration

$$\int_0^1 \int_{\sqrt{y}}^1 e^{x^3}\, dx\, dy.$$

Problem 2. [*10 points.*]

Find the critical points of the function

$$f(x, y) = xy^2 - 4xy + \frac{1}{2}x^2$$

and determine their nature.

Problem 3. [*7 points; 3, 3, 1.*]

Consider the function

$$f(x, y) = \frac{2x^2 y}{x^4 + y^2}.$$

(i) Show that the limit

$$\lim_{(x,y)\to(0,0)} f(x, y)$$

along any line of fixed slope m through the origin equals 0.

(ii) Evaluate the limit $\lim_{(x,y)\to(0,0)} f(x, y)$ along the parabola $y = x^2$.

(iii) What is the value of the limit $\lim_{(x,y)\to(0,0)} f(x, y)$?

Problem 4. [*13 points; 2, 2, 2, 3, 4.*]

Consider the function

$$f(x, y) = \sqrt{6 - x^2 - y^2}.$$

(i) Find the direction of steepest increase of f at the point $P(1, 2)$.

(ii) Draw the graph of the function f.

(iii) Find the directional derivative $D_{\vec{v}} f(1,2)$ in the direction $\vec{v} = \frac{3}{5}\vec{i} - \frac{4}{5}\vec{j}$.

(iv) Find the linear approximation $f((1,2) + .01 \cdot \vec{v})$.

(v) Find the tangent plane to the surface

$$z^2 x + z f(x^2, y) = 2$$

at $(1,2,1)$.

Problem 5. [*10 points.*]

Find the minimum and the maxim value of the function

$$f(x, y, z) = 2x - 2y + z$$

along the sphere of center $(1, 0, -1)$ and radius 3.

Problem 6. [*13 points.*]

Find the area of the region in the first quadrant bounded by the hyperbola $xy = 1$ and the parabolas $x = y^2$ and $x = 8y^2$. Express the answer in the simplest possible form.

Problem 7. [*13 points.*]

Find the average value of the function

$$f(x, y, z) = xyz$$

over the first octant

$$x \geq 0, y \geq 0, z \geq 0$$

of the ball $x^2 + y^2 + z^2 \leq 1$.

Problem 8. [*13 points.*]

Using cylindrical coordinates, find the mass of the solid in the upper half space $z \geq 0$ with density $\rho = z$ bounded by the sphere $x^2 + y^2 + z^2 = 2$ and the cone $z^2 = x^2 + y^2$.

Problem 9. [*10 points.*]

Find the volume of the solid bounded by the cylinders

$$z = 1 - y^2, \; z = y^2 - 1$$

and the planes $x = 0$ and $x + z = 1$.

Problem 10. [*12 points; 7, 5.*]

Consider the parametric curve given by

$$\vec{r}(t) = \left(\frac{t^4}{4}, \frac{t^6}{6}\right), \quad 0 \leq t \leq 1.$$

(i) Calculate the speed and the arclength function.

(ii) Find the arclength parametrization of the curve.

Problem 11. [*13 points.*]

Assume that

$$w = \ln(x^2 - y^2 + z^2)$$

where

$$x = 2s + t,\ \ y = 2s - t,\ \ z = 2\sqrt{st}.$$

Using the chain rule, calculate the derivative

$$\frac{\partial w}{\partial s}.$$

Express your answer in the simplest possible form.

Problem 12. [*8 points.*]

The ellipsoid $x^2 + 2y^2 + z^2 = 4$ and the plane $2x + y + 3z = 6$ intersect in an ellipse passing through the point $(1, 1, 1)$. Find the parametrization of the tangent line to the ellipse at $(1, 1, 1)$.