Math 20C - Fall 2011 - Midterm I

Name: _____

Student ID: _____

Section time: _____

Instructions:

Please print your name, student ID and section time.

During the test, you may not use books, calculators or telephones. You may use a "cheat sheet" of notes which should be at most half a page, front and back.

Read each question carefully, and show all your work. Answers with no explanation will receive no credit, even if they are correct.

There are 6 questions which are worth 50 points. You have 50 minutes to complete the test.

Question	Score	Maximum
1		8
2		8
3		6
4		5
5		15
6		8
Total		50

Problem 1. [*8 points; 5, 3*]

Consider the points $P(1, 1, -2)$, $Q(2, 0, 1)$ and $R(1, -1, 0)$.

(i) Find the area of the triangle PQR.

(ii) Find the equation of the plane through P, Q and R.

Problem 2. [*8 points; 4, 4.*]

(i) Find the constant a such that the function

$$f(x,y,z) = \begin{cases} \frac{x^4 y}{x^2+y^2+z^2} & \text{if } (x,y,z) \neq (0,0,0) \\ a & \text{if } (x,y,z) = (0,0,0) \end{cases}$$

is continuous.

(ii) Determine the following limit or explain why it does not exist

$$\lim_{(x,y)\to(0,1)} \frac{x^2(y-1)^2}{x^4+(y-1)^4}.$$

Problem 3. [*6 points.*]

Let $\vec{x}$, $\vec{y}$ and $\vec{z}$ be vectors whose magnitudes are 1, 2, and 1 respectively. Suppose that $\vec{x}$ is parallel to (and in the same direction as) $\vec{y}$, and $\vec{x}$ is perpendicular to $\vec{z}$. Find the angle between the vectors $\vec{x} + \vec{z}$ and $\vec{y} + 3\vec{z}$.

Problem 4. [*5 points.*]

A line ℓ is perpendicular to the plane $x + 2y - 3z = 2$ and passes through the point $(1, 0, -1)$. Where does the line ℓ intersect the plane $x - 2y + z = -4$?

Problem 5. [*15 points; 4, 3, 4, 4.*]

The trajectory of a particle is given by the parametric curve

$$x = 2\cos t + \sin t, \quad y = \cos t - 2\sin t, \quad z = 2t, \quad 0 \leq t \leq \pi.$$

(i) Find the velocity and speed of the particle.

(ii) Find the equation of the tangent line to the trajectory at the point $t = \frac{\pi}{2}$.

(iii) Find the arclength parametrization of the trajectory.

(iv) Show that the particle never leaves the surface of a fixed cylinder whose central axis is the z-axis. Carefully draw the trajectory of the particle.

Problem 6. [*8 points; 4, 4.*]

Consider the function $f(x, y) = 1 - \frac{1}{9}(x-1)^2 - \frac{1}{4}y^2$.

(i) Carefully sketch the level diagram for f showing three different levels.

(ii) Carefully sketch the graph of f.