Math 20C - Fall 2011 - Midterm II

Name: ______________________________

Student ID: ______________________________

Section time: ______________________________

Instructions:

Please print your name, student ID and section time.

During the test, you may not use books, calculators or telephones. You may use a ”cheat sheet” of notes which should be at most half a page, front and back.

Read each question carefully, and show all your work. Answers with no explanation will receive no credit, even if they are correct.

There are 5 questions which are worth 45 points. You have 50 minutes to complete the test.

Question	Score	Maximum
1		7
2		9
3		8
4		9
5		12
Total		45

Problem 1. [*7 points.*]

Find the volume of the region bounded above by the elliptic parabolid $z = 4 - x^2 - 3y^2$, on the bottom by the (x, y)-plane, on the sides by the planes $x = 0$, $x = 1$, $y = -1$ and $y = 1$.

Problem 2. [*9 points.*]

Find the critical points of the function

$$f(x, y) = x^3 + 6xy + 3y^2 - 9x$$

and determine their nature.

Problem 3. [*8 points.*]

Let $z = f(x, y)$ where f is a function such that

$$\frac{\partial f}{\partial y} = x - 2y.$$

It is furthermore known that

$$\frac{\partial f}{\partial x}(4, 3) = 2.$$

Assume that

$$x = \frac{u^2}{v}, \; y = 2uv - v^2.$$

Calculate the derivative

$$\frac{\partial z}{\partial v}$$

at the point $u = 2$ and $v = 1$.

Problem 4. [*9 points.*]

Find the minimum and the maximum value of the function

$$f(x, y) = x^2 + 2y^2 - 6x + 2$$

along the ellipse

$$2x^2 + y^2 = 8.$$

Problem 5. [*12 points; 3, 2, 3, 4.*]

Consider the function

$$f(x, y) = x^2y^4 + xy^2 \ln(2x - y).$$

(i) Find the direction of steepest increase for the function f at the point $(1, 1)$.

(ii) Find the directional derivative $D_{\vec{v}}f(1, 1)$ in the direction $\vec{v} = \frac{1}{\sqrt{2}}\vec{i} - \frac{1}{\sqrt{2}}\vec{j}$.

(iii) Find the tangent plane to the graph of f at the point $(1, 1, 1)$.

(iv) Find the tangent plane to the level surface $z^2x^3 - f(x, y) = 0$ at the point $(1, 1, 1)$.