
For this problem set, you may assume both Little and Great Picard.

1. If $h$ is meromorphic in $\mathbb{C}$, and omits three values then $h$ is constant.
   
   Hint: If $h$ omits $a, b, c$, what values does $\frac{1}{h-a}$ omit?

2. Let $n \geq 3$. If $f, g$ are entire such that $f^n + g^n = 1$, show that $f, g$ are constant.
   
   Hint: Find $n$ values that $f/g$ omits.

3. Let $f, g$ be two nonconstant entire functions, $P, Q$ two nonconstant polynomials such that
   
   $$e^f + P = e^g + Q.$$
   
   Show that $P = Q$.
   
   Hint: Consider $P - Q = e^g(1 - e^{f-g})$ and examine $1 - e^{f-g}$.

4. If $h$ is a nonconstant polynomial and $f$ is a nonconstant entire function, show that $he^f$ does not omit any values.

5. Let $f$ be entire such that $f \circ f$ has no fixed points. Show that $f(z) = z + a$ for some $a$.

   Hint: Let $g(z) = \frac{f(f(z)) - z}{f(z) - z}$. Show that $g$ omits the values 0 and 1, hence it is constant.
   
   Taking derivatives in $f(f(z)) - z = c(f(z) - z)$, show that $f' \circ f$ omits two values. Show $f'$ is constant and conclude.