Math 31CH - Spring 2013 - Midterm I

Name: ____________________________________________

Student ID: _______________________________________

Instructions:

Please print your name, student ID.

During the test, you may not use books, calculators or telephones.

Read each question carefully, and show all your work. Answers with no explanation will receive no credit, even if they are correct.

There are 5 questions which are worth 45 points. You have 50 minutes to complete the test.

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Problem 1. [5 points.]

Using cylindrical coordinates, set up a triple integral giving the volume of the solid contained between the two paraboloids

\[ z = x^2 + y^2, \quad z = 2 - x^2 - y^2. \]

You do not need to evaluate the integral.
**Problem 2.** [7 points.]

Consider a ball of radius 1 in $\mathbb{R}^3$:

$$x^2 + y^2 + z^2 \leq 1.$$ 

Find the average value of the distance $PO$ from a point $P$ inside ball to the center $O$ of the ball.
Problem 3. [11 points.]

A certain torus \( S \subset \mathbb{R}^4 \) admits the parametrization \( \gamma : [0, 2\pi) \times [0, 2\pi) \rightarrow S \) given by

\[
\gamma \left( \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \right) = \begin{bmatrix}
2\sin \alpha - \cos \alpha \\
\sin \alpha + 2\cos \alpha \\
\sin \beta \\
\cos \beta 
\end{bmatrix}.
\]

Find the surface area of the torus.
Problem 4. [11 points.]

Some of the entries of the following $3 \times 3$ matrix have been erased

$$A = \begin{bmatrix} 1 & \ast & \ast \\ 0 & 3 & \ast \\ 0 & \ast & -1 \end{bmatrix}.$$ 

However, it is known that the determinant of $A$ equals 1.

(i) [3] Verify that $\lambda_1 = 1$ is an eigenvalue for $A$ with eigenvector $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

(ii) [3] Find the remaining two eigenvalues $\lambda_2$ and $\lambda_3$.

*Hint:* Use the determinant and the trace of $A$. 
(iii) [2] Is the matrix $A$ diagonalizable? Why or why not?

(iv) [3] Find the determinant of the matrix $A + 2I$. 
Problem 5. [11 points.]

Using change of variables, calculate the mass of a plate with density $\delta(x, y) = x$ bounded by the curves

$$x^2y = 1, \quad x^2y = 2, \quad x = y, \quad x = 2y.$$