REVIEW PROBLEMS FOR MIDTERM I

First, review all the homework problems. In addition, you may want to solve the following problems. Solutions for the textbook problems can be found in the solution manual.


A. The characteristic polynomial of a square matrix is

\[ \chi_A(t) = t^3 - 3t^2 + at \]

for some unknown constant \( a \). In addition, it is known that \( A - 2I \) is not invertible.
  
(i) Show that \( A \) is not invertible.
  
(ii) Calculate the determinant of \( A + I \). You will need to first find the value of \( a \).
  
(iii) Is \( A \) diagonalizable?

B. Find the eigenvalues of the matrix

\[
A = \begin{bmatrix}
2 & 1 & 1 \\
1 & 2 & 1 \\
1 & 1 & 2
\end{bmatrix}.
\]

Is the matrix diagonalizable? What is the volume of the parallelogram spanned by the columns of \( A \)?

C. Assume that \( A \) is a symmetric matrix such that \( A^{2013} = I \). Show that \( A = I \).

(2) Integration in rectangular coordinates. Fubini’s theorem. Changing order of integration.

A. Evaluate by changing the order of integration

\[
\int_0^1 \int_{\sqrt{y}}^1 e^x \, dx \, dy.
\]

B. Find the area of the region in the first quadrant bounded by the hyperbola \( xy = 1 \) and the parabolas \( x = y^2 \) and \( x = 8y^2 \). Express the answer in the simplest possible form.

C. Find the volume of the solid bounded by the cylinders

\[ z = 1 - y^2, \quad z = y^2 - 1 \]

and the planes \( x = 0 \) and \( x + z = 1 \).

(3) Polar, cylindrical, spherical coordinates.

A. Using cylindrical coordinates, find the mass of the solid in the upper half space \( z \geq 0 \) with density \( \rho = z \) bounded by the sphere \( x^2 + y^2 + z^2 = 2 \) and the cone \( z^2 = x^2 + y^2 \).
B. Using spherical coordinates, find the volume of the cap of the spherical cap that lies between the sphere
\[ x^2 + y^2 + z^2 \leq 2 \]
and above the plane \( z = 1 \).

C. From the textbook, solve problem 4.10.9

(4) *Arbitrary changes of variables.*

A. Calculate the mass of a plate with density \( \delta(x, y) = xy \) contained between the ellipses
\[ x^2 + \frac{y^2}{4} = 2, \quad x^2 + \frac{y^2}{4} = 4 \]
and the hyperbolas \( x^2 - y^2 = 1 \) and \( x^2 - y^2 = 2 \).

B. Consider the region \( D \) bounded by the curves
\[ 1 \leq x^{30} + y^{60} \leq 2, \quad y^2 \leq x \leq 2y^2. \]
Integrate the function \( f(x, y) = \frac{x}{y^3} \) over \( D \).

C. From the textbook, solve problem 4.10.19.

(5) *Integration over arbitrary manifolds.*

A. From the textbook, solve problem 5.3.1.

B. From the textbook, solve problem 5.3.15(b).

C. From the textbook, solve problem 5.3.21.