Math 203 - Algebraic Geometry

Instructor: Dragos Oprea, doprea@math.you-know-where.edu.

Office hours: Room 6-101, Thursday 3-5pm (tentatively).

Textbook: Robin Hartshorne - Algebraic Geometry. The textbook is on reserve.

Lectures: MWF (10am-10:50pm), 7-421.

Webpage: http://math.ucsd.edu/~ doprea/203.html.

Goals: This course provides an introduction to algebraic geometry. Algebraic geometry is a central subject in modern mathematics, and an active area of research. It has connections with number theory, differential geometry, symplectic geometry, mathematical physics, string theory, representation theory, combinatorics and others.

Math 203 is a three quarter sequence. Math 203a will serve as preparation for a course in scheme theory (which may be covered in Math 203bc). Math 203bc will be taught by Professor Mark Gross in the Winter and Spring quarters.

We will study affine and projective algebraic varieties, and their properties. The goal is to cover roughly the first chapter (+epsilon) of Hartshorne's book. I hope to illustrate the general theory with many examples.

Syllabus. We will tentatively cover the following topics:

1. (i) Affine space and affine sets. Hilbert's Nullstellensatz. The correspondence between ideals and affine sets. Zarisky topology. Irreducible affine sets. Dimension.

(ii) Functions on affine varieties. Coordinate rings. Sheaves. Morphisms. Isomorphisms. Rational and birational maps. Fibered products.

2. (i) Prevarieties. Gluing. Projective space, projective varieties. Examples including hypersurfaces, quadrics, Grassmannians, elliptic curves.

(ii) Homogeneous coordinate rings. Morphisms. Examples including Segre embeddings, Veronese embeddings. Rational varieties.

3. Tangent spaces. Smoothness. Blowups. Dimension. If time allows: the 27 lines on a smooth cubic surface.

4. Intersections in projective space, intersection multiplicities. Bezout's theorem. Applications of Bezout e.g. Pascal's mystic hexagon. The addition law on cubic curves.

5. Smooth curves. If time allows, more on elliptic curves e.g. cubic curves don't have rational parametrization. Lattices. The Weierstrass function. Rational points.

Prerequisites: Some knowledge of modern algebra at the level of Math 200 is required. However, I will not assume background in commutative algebra. Familiarity with complex analysis, basic point set topology, differentiable manifolds is helpful, but not required. Since it is hard to determine the precise background needed for this course, I will be happy to discuss prerequisites on an individual basis. If you are unsure, please don't hesitate to contact me.

Problem Sets: There will be (weekly) problem sets, usually due on Friday. The problem sets will be posted online. Group work is encouraged, but you have to hand in your own write up of the homework problems. Late problem sets will not be accepted.

Final Grades: The final grades are based entirely on the homework.

Important dates: Drop deadline: October 10. Withdrawal deadline: October 24. Last day of classes: December 5.