From the textbook solve the following:

1. Page 181, problem 2.3.4(b), 2.3.5(b), 2.3.6.

2. 
   (i) Find two invertible matrices whose sum is not invertible.
   (ii) Give examples of matrices $A$ and $B$ such that
   \[ \det(A + B) \neq \det A + \det B. \]
   (iii) Exhibit two matrices $A$ and $B$ for which \( \det(A + B) = \det A + \det B. \)

3. For what values of $k$ is the matrix
   \[
   \begin{pmatrix}
   1 & 1 & 1 \\
   1 & 2 & k \\
   1 & 4 & k^2
   \end{pmatrix}
   \]
   invertible?

4. Calculate the determinant of the following matrices and find their inverses if they exist:
   (i) \[
   \begin{pmatrix}
   1 & 1 & 1 & 1 \\
   2 & 1 & 1 & 2 \\
   0 & 1 & 2 & 4 \\
   3 & 3 & 4 & 5
   \end{pmatrix}
   \]
   (ii) \[
   \begin{pmatrix}
   1 & 1 & 2 & 1 \\
   0 & 1 & 3 & 2 \\
   2 & -1 & 0 & 4 \\
   1 & 1 & 1 & 1
   \end{pmatrix}
   \]

5. I. Suppose that $C$ and $A$ are $n \times n$ matrices.
   (i) If $B = C^{-1}AC$ show that $CBC^{-1} = A$.
   (ii) Show that $C^{-1}A^mC = (C^{-1}AC)^m$ for all $m \geq 1$.

II. Consider
\[
A = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}.
\]
Calculate $CAC^{-1}$, and using part I, find $A^m$ for all values of $m$.

6. The trace of a square matrix is the sum of the elements on the main diagonal:
\[ \text{Tr } A = \sum_{i=1}^{n} a_{ii}. \]

Prove
(i) For any $2 \times 2$ matrix, we have

$$A^2 - \text{Tr} (A)A + \det A \cdot I = 0.$$ 

(ii) For any $n \times n$ matrices $A$ and $B$ we have

$$\text{Tr} AB = \text{Tr} BA.$$

7.

(i) Suppose $R$ is a quadrilateral with vertices $(0, 0)$, $(2, 1)$, $(3, -3)$ and $(4, -1)$. Find the area of $R$.

(ii) If $T$ is a linear transformation with matrix

$$\begin{bmatrix}
-3 & 2 \\
-1 & 2
\end{bmatrix}$$

find the area of the region $T(R)$.
