
From the textbook, solve the following questions:

1. (before Wednesday) For what values of $a$ and $b$ are the matrices
   
   \[
   A = \begin{bmatrix}
   0 & 0 & a \\
   1 & 0 & b \\
   0 & 1 & 0
   \end{bmatrix}
   \quad \text{and} \quad
   B = \begin{bmatrix}
   1 & 0 & 0 \\
   0 & 1 & 0 \\
   0 & 0 & -2
   \end{bmatrix}
   \]
   
   similar?

2. (Wednesday, Nov 24) Find an orthonormal eigenbasis for the matrices
   
   (i) $A = \begin{bmatrix}
   1 & -2 \\
   -2 & 4
   \end{bmatrix}$
   
   (ii) $A = \begin{bmatrix}
   1 & 0 & 0 & 1 \\
   0 & 0 & 0 & 0 \\
   0 & 0 & 0 & 0 \\
   1 & 0 & 0 & 1
   \end{bmatrix}$

3. (Wednesday, Nov 24) Show that if $A$ is a real symmetric matrix, $\det(A^2 + I) \neq 0$. (Diagonalize!)

4. (Wednesday, Nov 24) Find all symmetric matrices $A$ such that $A^{2011} = I$. (Diagonalize! What are the eigenvalues of $A$?)

5. (Monday, Nov 29) Find the matrix of each quadratic form and determine the definiteness of the form:
   
   (i) $Q(x, y) = 3x^2 - 2xy + 2y^2$
   
   (ii) $Q(x, y, z, w) = x^2 + 4yz$
   
   (iii) $Q(x, y, z, w) = -x^2 - 2y^2 - z^2$

6. (Monday, Nov 29)
   
   (i) Show that if $Q_1, Q_2$ are quadratic forms with matrices $A_1$ and $A_2$, $Q_1 + Q_2$ is a quadratic form with matrix $A_1 + A_2$.
   
   (ii) Using (i), show that if $A_1$ and $A_2$ are symmetric matrices with only positive eigenvalues, $A_1 + A_2$ has only positive eigenvalues.

7. (Monday, Nov 29) Discuss the definiteness of the quadratic form with matrix
   
   $A = \begin{bmatrix}
   1 & 0 & 2 \\
   0 & 1 & 0 \\
   2 & 0 & 1
   \end{bmatrix}$.