Name: ________________________________

Student ID: __________________________

Instructions:

Please print your name and student ID.

During the test, you may not use books or notes.

Read each question carefully, and show all your work. Answers with no explanation will receive no credit, even if they are correct.

There are 4 questions which are worth 35 points, and a bonus question. You have 50 minutes to complete the test.

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<th>Question</th>
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Problem 1. [8 points]

(i) [4] Find the inverse of the matrix

\[
A = \begin{bmatrix}
1 & -1 & 1 \\
-1 & 1 & 0 \\
0 & 1 & 1
\end{bmatrix}.
\]
(ii) [2] Possibly using your solution to (i) (or any other method) find the determinant of $A$.

(iv) [2] Is $A$ the matrix of a projection onto a plane $V \subset \mathbb{R}^3$? If ‘yes’, which plane? If ‘no’, why not?
Problem 2. [7 points]

Consider line $L$ spanned by the vector \[
\begin{bmatrix}
1 \\
1 \\
0 \\
1
\end{bmatrix}
\] in $\mathbb{R}^4$.

(i) [4] Find the matrix of the orthogonal projection onto $L$.

(ii) [3] Find the matrix of the orthogonal projection onto $L^\perp$. 
Problem 3. [12 points]

Consider the subspace $V \subset \mathbb{R}^4$ spanned by the vectors
\[
\begin{bmatrix}
1 \\
1 \\
1 \\
0
\end{bmatrix}
\quad \text{and} \quad
\begin{bmatrix}
-1 \\
-2 \\
0 \\
1
\end{bmatrix}.
\]

(i) [4] Give a basis of $V^\perp$. 

(ii) [4] Find an orthonormal basis for the subspace $V$.

(iii) [4] Find the projection of the vector $\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ onto the subspace $V$. 
Problem 4. [8 points.]

True or false:

T F For a square $n \times n$ matrix $A$, $\dim N(A) + \dim C(A) = n$.

T F For a square $n \times n$ matrix $A$, $\dim N(A) + \dim C(A^T) = n$.

T F An orthogonal $n \times n$ matrix is invertible.

T F For any matrix $A$, $A^T A$ is invertible.

T F If $\det A = 2$ and $\det B = 3$ then $\det (A + B) = 5$.

T F The product of two invertible matrices may not be invertible.

T F If $A$ is an orthogonal matrix corresponding to a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, and $R$ is a region in $\mathbb{R}^2$, then the area of $T(R)$ equals the area of $R$.

T F Exchanging two columns of a square matrix changes the sign of the determinant.
Extra Credit [5 points]

Assume that $A$ is a $2 \times 2$ matrix such that $A^n = 0$ for some $n \geq 0$. Show that $A^2 = 0$. 