RECOMMENDED PROBLEMS - FINAL EXAM

1. Point Set Topology

Prove rigourously that the set of matrices where $A + A^2$ is invertible is open in $\text{Mat}_{n \times n}$.

2. Limits

Problem 1.19, page 158.

3. Continuity

Using $\varepsilon$ and $\delta$s, show that if $f$ and $g$ are continuous real valued functions, then $f + g$ and $fg$ are also continuous.

4. Directional derivatives, gradient

Consider the function $f(x, y) = e^{-3x+2y}\sqrt{2x+1}$.

(i) Calculate the gradient of $f$ at $(0, 0)$.
(ii) Find the directional derivative of $f$ at $(0, 0)$ in the direction $u = \frac{i + j}{\sqrt{2}}$.
(iii) What is the unit direction for which the rate of increase of $f$ at $(0, 0)$ is maximal? What is the rate of increase?

5. Pathological functions

Problem 1.33 page 159.

6. Taylor polynomials

Problem 3.7, page 390.

7. Chain rule

Assume that $u$ and $v$ are harmonic conjugates and that $f$ and $g$ are also harmonic conjugates. Show that $F(x, y) = u(f(x, y), g(x, y))$, $G(x, y) = v(f(x, y), g(x, y))$ are also harmonic conjugates.

8. Functions of matrices

Problem 1.31 page 159.

9. Critical points and second derivative test

Problem 3.19, page 391.

10. Functions on compact sets.

Find the global minimum and global maximum of the function $f(x, y, z) = x^2 + y^2 + z^2 - 2x - 4y - 6z$ over the compact set $x^2 + y^2 + z^2 \leq 15$. 

11. Lagrange multipliers

Problem 3.22, page 392.

12. Inverse function theorem

Problem 2.30, page 282.

13. Implicit function theorem

Problem 3.11(a), page 390. Also calculate the derivative $D_1 g_r(r,0)$.

14. Manifolds

Problem 3.1, page 389.

15. Tangent spaces

Problem 3.4, page 389.