1. (Wednesday, February 9.) Pathological functions. From the textbook, solve 1.9.1 and 1.9.2.

2. (Wednesday, February 9.) Pathological functions and second order derivatives. Put

\[ f(x, y) = \begin{cases} 
  xy \cdot \frac{x^2 - y^2}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\
  0 & \text{if } (x, y) = (0, 0).
\]

(i) Calculate the first derivative \( f_x \) at \((0, 0)\) directly from the definition.

(ii) Calculate the first derivative \( f_x \) at any other point \((x, y) \neq (0, 0)\).

(iii) Differentiate once more i.e. calculate \( f_{xy}(0, 0) \) using the definition. Confirm that \( f_{xy}(0, 0) = 1 \).

(iv) Repeat for the derivative \( f_y(0, 0) \) then \( f_{yx}(0, 0) \). Confirm that \( f_{yx}(0, 0) = -1 \). Observe that \( f_{xy}(0, 0) \neq f_{yx}(0, 0) \).

3. (Monday, February 14.) From the textbook solve 3.6.1, 3.6.2, 3.6.7.

4. (Monday, February 14.) Two space shuttles are at the points \((1, 0, -1)\) and \((6, 1, 0)\) at time \(0\) and are travelling in straight lines parallel to the vectors \(-2i + j\) and \(4i - j - k\). What is the minimum distance between the two paths and when is it achieved?

(a) Write the parametric equations for the two paths. Use two different parameters \(t\) and \(u\) for each of the lines. Determine the (square) of the distance \(d(t, u)\) between two points \(A\) and \(B\) on each of the lines. Now minimize the square distance. Find the corresponding \(t_0, u_0\) minimizing the distance and also find the points where the distance is minimal. As the space shuttles travel, are they at some moment in time this minimal distance?

(b) Verify using the second derivative test that the distance you found in part (a) is indeed a minimum for \(d(t, u)\) and not a maximum or a saddle point.