

Math 52 - Fall 2007 - Midterm II

Name: \_\_\_\_\_

Student ID: \_\_\_\_\_

Signature: \_\_\_\_\_

**Instructions:**

Please print your name and student ID. Your signature indicates that you accept the honor code.

During the test, you may not use notes, books, calculators or telephones. Read each question carefully, and show all your work.

Question	Score	Maximum
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
Total		80

**Problem 1.**

Find the simple positively oriented closed curve  $C$  across which the flux of the field

$$\mathbf{F} = (x^4y - x^3)\mathbf{i} + (y - y^3 - 2x^3y^2)\mathbf{j}$$

is maximal. What is the maximal value of the flux?

**Problem 2.**

Let  $C$  be the curve parametrized as

$$x(t) = 2 \cos t, \quad y(t) = 3 \sin t \quad \text{for } 0 \leq t \leq \pi.$$

Find the value of the line integral

$$\int_C (\cos x + y^2 \sin x) dx + (\sin y - 2y \cos x) dy.$$

**Problem 3.**

(i) Find a potential for the field

$$\mathbf{F} = (1 + ye^{xy})\mathbf{i} + (2y + xe^{xy})\mathbf{j}.$$

(ii) Consider the field

$$\mathbf{G} = (ax^2 + xy^2)\mathbf{i} + (xy + by^3)\mathbf{j}.$$

Find the constants  $a$  and  $b$  such that the flux

$$\int_C \mathbf{G} \cdot \mathbf{n} \, ds$$

is path independent (i.e. it depends only the endpoints of the simple oriented path  $C$ .)  
Explain your answer.

**Problem 4.**

Compute the line integral

$$\int_C \frac{x}{x^2 + y^2} dx + \frac{y}{x^2 + y^2} dy$$

over any positively oriented simple closed curve  $C$ . You may assume that the origin does not lie on the curve  $C$ .

**Problem 5.**

Compute the gravitational attraction on a unit mass particle placed at the origin from the solid contained below the plane  $z = 1$  and above the right circular cone  $z^2 = x^2 + y^2$ . Take the density to be the distance from the vertex.

**Problem 6.**

Using cylindrical coordinates, determine the volume of the solid bounded above by the sphere of radius  $\sqrt{2}$  centered at the origin, and below by the paraboloid  $z = x^2 + y^2$ .

**Problem 7.**

Find the volume bounded by the elliptic paraboloid  $y = x^2 + 4z^2$  and the plane  $y = 2x + 3$ .

**Problem 8.**

Consider the field

$$\mathbf{F} = z\mathbf{k}.$$

Let  $S$  be the closed surface bounded below by the cone  $z^2 = x^2 + y^2$ , and bounded above by the sphere of radius 2 centered at the origin. Check that the divergence theorem is satisfied for the surface  $S$  and the field  $\mathbf{F}$ .