Math 52, Problem Set 3. Due Wednesday, Oct 17.

1. (`Thursday-Friday: Line integrals`)

Page 379, problems 14, 18, 20, 27, 29.

2. (`Thursday-Friday: Line integrals`)

Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where

$$\mathbf{F} = 2x\,\mathbf{i} + (x - y)\,\mathbf{j}$$

and C is the clockwise triangle with vertices $(0, 0)$, $(1, 0)$ and $(0, 1)$.

3. (`Thursday-Friday: Line integrals`)

Let $\mathbf{F} = \mathbf{3i} + \mathbf{2j}$. How would you place a unit segment C such that the value of

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

is maximum? How about minimum? What are the min/max values?

4. (`Thursday-Friday: Line integrals`)

Let C be the counterclockwise circle of radius a centered at the origin. Consider the fields

$$\mathbf{F} = x\,\mathbf{i} + y\,\mathbf{j}, \mathbf{G} = y\,\mathbf{i} - x\,\mathbf{j}.$$

Without parametrizing C, but using only the geometric properties of the fields, compute the line integrals

$$\int_C \mathbf{F} \cdot d\mathbf{r} \text{ and } \int_C \mathbf{G} \cdot d\mathbf{r}.$$

5. (`Monday-Tuesday: Gradient fields`)

Page 399, problems 4 and 6. You do not need to find a potential function in case we do not cover that topic in class.

6. (`Monday-Tuesday: Path independence, gradient fields`)

Consider the following field in the (x, y) plane:

$$\mathbf{F} = -\nabla(\ln r).$$

(a) Let C be any path joining two points P_1 and P_2 at distances r_1 and r_2 from the origin. Show that $\int_C \mathbf{F} \cdot d\mathbf{r}$ depends only on r_1 and r_2 but not on the path C.

(b) For what position of a line segment L of length 1 do we expect

$$\int_L \mathbf{F} \cdot d\mathbf{r} = 0.$$

(c) Compute the field $\mathbf{F}$ in rectangular coordinates.

(d) Compute the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where C is a circle of radius a centered at the origin. Use two different methods. First, use the results of part (a). Second, use an explicit computation in rectangular coordinates, using a convenient parametrization of the circle.

7. (`Monday-Tuesday: Path independence, gradient fields`)

Let $f(x, y) = x^4 y + y^2$, and C be the parabola $y^2 = x$, between $(1, -1)$ and $(1, 1)$, directed upwards. Calculate $\mathbf{F} = \nabla f$. Then determine the integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

in three different ways:

(i) directly;
(ii) by using path-independence to replace C by a simpler path;
(iii) by using the Fundamental Theorem for line integrals.

8. (`Monday-Tuesday: Gradient fields`)

Find the values of a, b for which the field $\mathbf{F} = (ax^2y - 3y - xy^2 + 1)\mathbf{i} + (x^3 - y - bx - x^2y)\mathbf{j}$ is conservative.