Math 52 - Winter 2007 - Midterm Exam II

Name: ______________________________

Student ID: ______________________________

Section number and TA name: ______________

Signature: ______________________________

Instructions: Print your name and student ID number, print your section number and TA's name, write your signature to indicate that you accept the honor code. During the test, you may not use notes, books or calculators. Read each question carefully, and show all your work. You have two hours (7PM-9PM) to answer all the questions.

Question	Score	Maximum
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total		100

Problem 1. (10 pts.) Let C_r be the upper semicircle $x^2 + y^2 = r^2$ with $y \geq 0$ oriented from left to right. Let $f(x)$ be any continuous function with a continuous derivative. Show that

$$\int_{C_r} (y \cdot f'(x))\ dx + (x + f(x))\ dy = -\frac{1}{2}\pi r^2$$

Hint: Consider the boundary of the half disk $x^2 + y^2 \leq r^2$ with $y \geq 0$.

Problem 2. (10 pts.) Let C be the curve parametrized by $\mathbf{r}(t) = \left(t{\cdot}e^{t^2}, \sin\left(2t^6 - t^3\right), \arctan\left(t^3\right)\right)$ for $0 \leq t \leq 1$. Find

$$\int_C y\,dx + x\,dy.$$

Hint: Use the Fundamental Theorem for Line Integrals.

Problem 3. (10 pts.) Compute the coordinates of the centroid of the curve that is the **boundary** of the triangle with the vertices $(0, 0)$, $(2, 0)$ and $(0, 5)$.

Problem 4. (10 pts.) Show that there is no vector field $\vec{F}$ such that

$$\nabla \times \vec{F} = \langle x, 0, 0 \rangle$$

Problem 5. ******** new problem ***********Fix two positive numbers a and b. Let C be the curve in the xy-plane consisting of two line segments which join $(0, 1)$ to $(a, 0)$ and then $(a, 0)$ to (a, b).

(a) (5 pts.) Evaluate

$$\int_C \frac{2x\,dx + 2y\,dy}{x^2 + y^2}$$

(the answer will depend on a and b).

(b) (5 pts.) Is the vector field

$$\vec{F} = \langle \frac{2x}{x^2 + y^2}, \frac{2y}{x^2 + y^2} \rangle$$

conservative?

Notice: $\vec{F}$ is not defined at the origin, so saying "$\nabla \times \vec{F} = \vec{0}$" IS NOT ENOUGH.

Problem 6. (10 pts.) Find the value of the integral

$$\iint\limits_{\mathrm{S}} xyz\, dS$$

over the surface that is the boundary of the rectangular solid cut from the first octant by the planes $x = a$, $y = b$ and $z = c$.

Problem 7. (10 pts.) Let Γ be a curve parametrized by

$$\gamma(t) = (t,\ 0,\ f(t))$$

for some smooth function $f(t)$ and $0 < a \leq t \leq b$.
(a) Parametrize the surface S obtained by rotating the curve Γ about the z axis.

(b) Indicate the order of variables in the above parametrization that is needed for S to be oriented with the normal vector pointing up.

(c) Setup, but DO NOT EVALUATE an integral representing the surface area of S.

Problem 8. (10 pts.) Let S be the cylinder of radius a, along the x-axis, and bounded by the two planes $x = 0$ and $x = b$.

1. Find a surface parametrization X for S, for which the normal vector points outwards.

2. Compute $\iint_S F \cdot dS$, where S is oriented such that the normal vector points outwards, and $F(x, y, z) = (\log(x + 1), y, z)$.