

Math 10C - Fall 2009 - Midterm I

Name: \_\_\_\_\_

Student ID: \_\_\_\_\_

Section time: \_\_\_\_\_

**Instructions:**

Please print your name, student ID and section time.

During the test, you may not use books or telephones. You may use a "cheat sheet" of notes which should be a page, front only.

Read each question carefully, and show all your work. Answers with no explanation will receive no credit, even if they are correct.

There are 5 questions which are worth 55 points. You have 50 minutes to complete the test.

Question	Score	Maximum
1		8
2		10
3		10
4		12
5		15
Total		55

**Problem 1.** [8 points.]

Consider the vectors

$$\mathbf{v} = \mathbf{i} + 2\mathbf{j}, \quad \mathbf{w} = 3\mathbf{i} - \mathbf{j}.$$

(i) [3 points] Draw the two vectors  $\mathbf{v}$  and  $\mathbf{w}$  in standard position, and draw their sum  $\mathbf{v} + \mathbf{w}$ .

(ii) [2 points] Compute the vector  $2\mathbf{v} + 3\mathbf{w}$ .

(iii) [3 points] Find the unit vector  $\mathbf{u}$  in the direction of  $\mathbf{w}$ .

**Problem 2.** [10 points.]

Consider the function

$$f(x, y) = 1 - (x - 1)^2 - (y - 1)^2.$$

(i) [3 points] Draw the contour diagram for  $f(x, y)$  and clearly label the level curves. Show the contours for at least three levels.

(ii) [4 points] Draw the graph of  $z = f(x, y)$ .

(iii) [3 points] Draw the cross sections of  $f(x, y)$  with  $x = 1$  and  $x = 2$ .

**Problem 3.** [10 points.]

- (i) [4 points] Suppose that  $z = f(x, y)$  is a linear function of  $x$  and  $y$ , with slope 3 in the  $x$  direction and slope  $-2$  in the  $y$  direction. A change of  $.1$  in  $x$  and  $-.2$  in  $y$  produces what change in  $z$ ?

- (ii) [6 points] The graph of a linear function  $z = g(x, y)$  passes through the point  $(1, 2, 5)$ . The graph intersects the  $xz$  plane along the line  $z = 3x + 6$ . Determine the linear function  $g$  and sketch its graph.

**Problem 4.** [12 points.]

(i) [3 points] Write the following sum in a simpler form

$$\frac{3}{2} + \frac{3}{2^2} + \frac{3}{2^3} + \dots + \frac{3}{2^{2009}}.$$

(ii) [3 points] Find the value of  $z$  for which

$$1 - z + z^2 - z^3 + \dots = \frac{3}{4}.$$

(iii) [3 points] Find the quadratic Taylor polynomial (around 0) for the function  $f(x) = \ln(x^2+1)$ .

(iv) [3 points] The third degree Taylor polynomial of a function  $f(x)$  equals

$$P_3(x) = 1 - \frac{x^2}{6} + \frac{x^3}{12}.$$

Is it true that 0 must be a local maximum of  $f$ ? Justify your answer.

**Problem 5.** [15 points.]

The outcome  $x$  of an experiment has the cumulative density function

$$f(x) = \begin{cases} c \left(1 - \frac{4}{x^2}\right) & \text{for } x \geq 2, \\ 0 & \text{for } x < 2. \end{cases}$$

(i) [3 points] Sketch the graph of  $f(x)$ . Show that we must have  $c = 1$ .

(ii) [3 points] Determine the probability density function.

(iii) [3 points] What is the probability that the outcome of the experiment is between 2 and 4?

(iii) [3 points] What is the median outcome of the experiment?

(iv) [3 points] Find the mean value for the outcome of the experiment.