Math 10C - Fall 2009 - Midterm II

Name: _____

Student ID: _____

Section time: _____

Instructions:

Please print your name, student ID and section time.

During the test, you may not use books or telephones. You may use a "cheat sheet" of notes which should be a page, front only.

Read each question carefully, and show all your work. Answers with no explanation will receive no credit, even if they are correct.

There are 5 questions which are worth 50 points. You have 50 minutes to complete the test.

Question	Score	Maximum
1		10
2		10
3		10
4		10
5		10
Total		50

Problem 1. [*10 points.*]

Consider the points P, Q, R with coordinates $(2, 1, 0)$, $(0, 1, 3)$ and $(1, 0, 1)$ respectively.

(i) Find the area of the parallelogram spanned by the vectors $\vec{PQ}$ and $\vec{PR}$.
(ii) Find the equation of the plane through P, Q, R.

Problem 2. [*10 points.*]

Find the second order Taylor polynomial near $(1, -1)$ for the function

$$f(x, y) = x^3 y.$$

Problem 3. [*10 points.*]

Consider the function

$$f(x, y) = x^4 y^3.$$

(i) [4] Write down the equation of the tangent plane at the graph of the function at the point $(1, 1, 1)$.

(ii) [4] Write down an expression for the change, Δz, in $z = f(x, y)$ depending on Δx and Δy, the change in x and y, respectively, near the point $x = y = 1$. Is the function $f(x, y)$ more sensitive to a change in x or to a change in y?

(iii) [3] Using your answer to (ii), find the approximate value of $f(1.01, 1.01)$.

Problem 4. [*10 points*]
Consider the function $f(x, y) = xe^{x+y}$ and the point $P = (2, -22)$.

(i) [4] Find the gradient of f at P.

(ii) [3] Find the directional derivative of f at P in the direction $\mathbf{u} = \frac{1}{\sqrt{2}}(\mathbf{i} - \mathbf{j})$.

(iii) [3] What is the direction of steepest increase for the function f at P? Express your answer as a unit vector.

Problem 5. [*10 points*]

Consider the function

$$w = \sin(xy)$$

where

$$x = \frac{1}{v}, \quad y = u^2 v.$$

Using the chain rule, calculate the derivatives

$$\frac{\partial w}{\partial u} \quad \text{and} \quad \frac{\partial w}{\partial v}.$$

Please express your answer in simplest form.