Problem 1.

In a coordinate system with origin O, consider the points P, Q, R with coordinates $(1,1,1)$, $(2,1,0)$ and $(a,2,3)$ respectively.

(a) For what value of the parameter a, will the vectors PQ and PR be perpendicular?
(b) When $a=3$, find the area of the triangle PQR.
(c) When $a=3$, find the equation of the plane through P, Q, R.

Problem 2.

Consider the function $f(x,y)=\frac{x}{y^2}$ and the point $P=(1,1)$.

(a) Sketch the level curve passing through P and contained in the first quadrant.
(b) Find the gradient of f at P.
(c) Find the directional derivative of f at P in the direction $\frac{1}{\sqrt{2}}(\mathbf{i}-\mathbf{j})$.
(d) Compute the differential of f.
(e) Give an approximation formula showing how small changes Δx and Δy produce a change Δz when $x=y=1$, for $z=f(x,y)$.
(f) Linearly approximate $f(1.1,.9)$.
(g) Find the second degree Taylor polynomial of f around $(1,1)$.

Problem 3.

(a) Find the tangent plane to the graph $z=x^2y^3+\ln(xy)$ at the point $(1,1,1)$.
(b) At the point $(1,1)$ in which direction (unit vector) does the value of the function $x^2y^3+\ln(xy)$ increase most rapidly? What is the rate of increase?

Problem 4.

(a) Let $x=u^2+v^2$ and $y=uv$ and let $f(x,y)$ be a differentiable function. Assume that at the point where $x=2$ and $y=1$, the gradient in the (x,y) coordinates is

$$\nabla f=\langle 1,1\rangle.$$

Calculate the partial derivatives

$$\frac{\partial f}{\partial u},\ \frac{\partial f}{\partial v}$$

at the point $u=v=1$.
(b) If $w=x\ln y, x=u+v, y=e^v$ compute the derivatives $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$.

Problem 5.

If

$$f_x(2,1)=-3, f_y(2,1)=4$$

find the tangent line to the level contour of f through the point $(2,1)$.

Answers:

(1.a) $a = 3$

(1.b) $\frac{3\sqrt{2}}{2}$

(1.c) $x - 4y + z = -2$

(2.a) *parabola* $x = y^2$.

(2.b) $\nabla f = (1, -2)$

(2.c) $f_u = \frac{3}{\sqrt{2}}$

(2.d) $df = \frac{1}{y^2} dx - \frac{2x}{y^3} dy$

(2.e) $\Delta z = \Delta x - 2\Delta y$

(2.f) $f(1.1, .9) \approx 1.3$

(2.g) $1 + (x-1) - 2(y-1) - 2(x-1)(y-1) + 3(y-1)^2$

(3.a) $3x + 4y - z = 6$

(3.b) $\left(\frac{3}{5}, \frac{4}{5}\right)$, $f_u = 5$.

(4.a) $f_u = v, f_v = u + 2v$

(4.b) $f_u = 3, f_v = 3$

(5) *the gradient is normal to the tangent line through the point* $(2, 1)$. *Thus for* (x, y) *on the tangent line,* $\nabla f \cdot (x-2, y-1) = 0 \implies -3(x-2) + 4(y-1) = 0 \implies -3x + 4y = -2$