

Math 20E Practice Test #1 (Driver)

Notation: Throughout this test $\mathbf{R} = (x, y, z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ will denote a position vector and $\mathbf{F}(\mathbf{R}) = F_1(\mathbf{R})\mathbf{i} + F_2(\mathbf{R})\mathbf{j} + F_3(\mathbf{R})\mathbf{k}$ will denote a vector field.

1. Captain Ralph is in trouble near the sunny side of Mercury. The temperature of the ship's hull when he is at location (x, y, z) will be given by $T(x, y, z) = \exp(-x^2 - 2y^2 - 3z^2)$, where x, y , and z are measured in meters. He is currently at $(1, 1, 1)$.
 - (a) In what direction should he proceed in order to decrease the temperature most rapidly?
 - (b) If the ship travels at e^8 meters per second, how fast will be the temperature decrease if he proceeds in that direction (degrees per second)?
 - (c) What is the rate of change of the temperature at $t = 0$ if the ship moves along a curve $\mathbf{R}(t) \in \mathbb{R}$ such that $\mathbf{R}(0) = (1, 1, 1)$, and $\dot{\mathbf{R}}(0) = \frac{d}{dt}\mathbf{R}(t)|_{t=0} = (-1, 2, -3)$.
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2. Find tangent plane to the surface S described by $z = \frac{1}{x^2+y^2}$ at $(1, 1, \frac{1}{2})$.
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3. Suppose that a small weather balloon is placed at $t = 0$ at location $(1, 1, 1)$ in a wind velocity field described by $\mathbf{F}(x, y, z) = (y + 1, 2, \frac{1}{2z})$. Show $\mathbf{R}(t) = ((t + 1)^2, 1 + 2t, \sqrt{t + 1})$ (for $t > 0$) describes the position of the balloon at later times.
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4. Given $\mathbf{F} = (x + xz^2)\mathbf{i} + xy\mathbf{j} + yz\mathbf{k}$ evaluate:
 - (a) $\nabla \cdot \mathbf{F}$
 - (b) $\nabla \times \mathbf{F}$.
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5. Compute the divergence and curl of the following vector fields.

$$V(x, y, z) = -y\mathbf{i} + x\mathbf{j}$$

$$W(x, y, z) = \frac{V(x, y, z)}{(x^2 + y^2)^{1/2}}$$

$$Y(x, y, z) = \frac{V(x, y, z)}{(x^2 + y^2)}.$$

6. Compute $\int_C \mathbf{F} \cdot d\mathbf{R}$ where $\mathbf{F}(x, y, z) = (xy, yz, xz)$ and C is the straight line path from $(0, 0, 0)$ to $(1, 1, 1)$.
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7. Suppose $\mathbf{F} = (F_1(x, y), F_2(x, y), 0)$ is a vector field which at $z = 0$ is pictured in Figure 1 below.

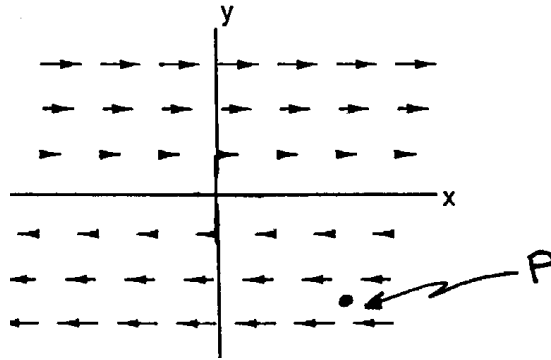


FIGURE 1. Graphical representation of \mathbf{F} when $z = 0$.

- (a) Say whether $\nabla \cdot \mathbf{F}(P)$ is positive, zero, or negative.
 (b) Say whether $\mathbf{k} \cdot \nabla \times \mathbf{F}(P)$ is positive, zero, or negative.
 Give reasons for your answer.

8. At what angle does the line $2x = y = z$ intersect the ellipsoid
 $2x^2 + 3y^2 + z^2 = 18$.