1. (a) FALSE. Existence of inverses; for example, $2^{-1}$.
   (b) FALSE. Must have same operation in group and subgroup.
   (c) TRUE.
   (d) FALSE. Odd times odd is even. Alternate: the identity is even.
   (e) FALSE. Give an example such as $(123)(45) \in S_5$, which has order 6.

2. (a) $\alpha = (152)(34)$
   (b) $|\alpha|$ is the least common of its cycle lengths when in disjoint form; that is, $\text{lcm}(3, 2) = 6$.
   (c) It is odd. The parity is the same as the parity of the number of even cycles.

3. 

<table>
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<tr>
<th>generator</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>order</td>
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<td>20</td>
<td>10</td>
<td>5</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

4. (a) Given two elements $X = axa^{-1}$ and $Y = aya^{-1}$ with $x, y \in H$, we have
   
   $$XY^{-1} = axa^{-1}(aya^{-1})^{-1} = axa^{-1}(ay^{-1}a^{-1}) = a(xy^{-1})a^{-1}.$$ 
   
   This is in $aHa^{-1}$ since $xy^{-1} \in H$ because $H$ is a group.
   (b) $\varphi(x)\varphi(y) = axa^{-1}aya^{-1} = axya^{-1} = \varphi(xy)$.