

- Setting  $dy/dt$  we find the equilibria  $y = -\sqrt{2}$ ,  $0$  and  $\sqrt{2}$ . Either by sketching  $dy/dt$  as a function of  $y$ , or by noting the sign of  $dy/dt$  on each side of an equilibrium, or by noting whether  $dy/dt$  is increasing (unstable) or decreasing (stable) near each equilibrium, we find that  $\pm\sqrt{2}$  are stable and  $0$  is unstable.
- Separate variables and integrate:  $\int e^{-y}dy = \int te^t dt$ . Thus  $-e^{-y} = te^t - e^t + C$ .
- We have  $r^2 - 2r + 2 = 0$  and so  $r = 1 \pm i$ . You can either leave the answer in complex form or use trig functions:

$$y = C_1 e^{(1+i)t} + C_2 e^{(1-i)t} \quad \text{or} \quad y = C_1 e^t \cos t + C_2 e^t \sin t.$$

- We have  $r^2 - 6r + 9 = 0$  and so  $r = 3$  is a double root. Thus  $y = C_1 e^{3t} + C_2 t e^{3t}$ .
- Since  $y' - y = 2te^t$ , an integrating factor is  $\mu = e^{\int -dt} = e^{-t}$ . Thus  $(e^{-t}y)' = 2t$  and so  $e^{-t}y = t^2 + C$ . You can leave it this way or solve for  $y$ :  $y = (t^2 + C)e^t$ , which is what you would have gotten using the formula for the solution to a linear first order equation.
- You can use the Wronskian method or the reduction of order method. We have  $y_1 = e^x$ .

- The Wronskian is  $Ce^{\int(1+x^{-1})dx} = Cxe^x$ . Since  $C$  is arbitrary, I'll let  $C = 1$ . Then  $e^x y_2' - e^x y_2 = xe^x$  and so  $y_2' - y_2 = x$ . An integrating factor is  $\mu = e^{-x}$  and so  $y_2 e^{-x} = \int xe^{-x} dx = -xe^{-x} + \int e^{-x} dx = -e^{-x}(1+x)$ . Thus  $y_2 = -(1+x)$  and so  $y = C_1 e^x + C_2(1+x)$ . (I included the minus sign in  $C_2$ .)

Alternatively, you can start with the Wronskian result in Problem 3.6.33:  $(y_2/y_1)' = W(y_1, y_2)/y_1^2$ , which gives  $y_2/y_1 = \int (xe^x)e^{-2x} dx = \int xe^{-x} dx$ .

- For reduction of order, we set  $y(x) = v(x)y_1(x) = e^x v(x)$  and obtain  $e^x v'' + (2e^x - (1 + 1/x)e^x)v' = 0$  and so  $v'' + (1 - 1/x)v' = 0$ . Separating variables or treating this as a linear equation,  $x^{-1}e^x v' = C$ . Choosing  $C = 1$ , we have  $v' = xe^{-x}$  and so  $v = -(x+1)e^{-x}$ . Thus  $y_2 = -(x+1)$ .

Other choices of the constants along the way will give the general solution in other forms. Regardless of how you do it, your final answer should have exactly two arbitrary constants.