The remarks are intended to enhance your understanding.
You do not have to hand in your answers to them.

11–16  Use the given transformation to evaluate the integral.

12. $\iint_{R}(4x + 8y)\,dA$, where $R$ is the parallelogram with vertices $(-1,3)$, $(1,-3)$, $(3,-1)$, and $(1,5)$;
   $x = \frac{1}{4}(u + v), \ y = \frac{1}{4}(v - 3u).
   $Remark: How does this improve the boundary of the region?

14. $\iint_{R}(x^2 - xy + y^2)\,dA$, where $R$ is the region bounded by the ellipse $x^2 - xy + y^2 = 2;
   x = \sqrt{2}u - \sqrt{2/3}v, \ y = \sqrt{2}u + \sqrt{2/3}v
   $Remark: What does this do to the boundary and to the integrand? Do you think this would still be a good idea if the integrand were $x^2 + y^2$?

16. $\iint_{R}y^2\,dA$, where $R$ is the region bounded by the curves $xy = 1, \ xy = 2, \ xy^2 = 1, \ xy^2 = 2;
   u = xy, \ v = xy^2.$ Illustrate the region $R$. (You may use a calculator.)

S1. Evaluate the improper integral $\iint x^2e^{-(x^2+y^2)}\,dA$ over the entire $xy$-plane.

S2. A sphere of radius $R$ is bounded by the surface $x^2 + y^2 + z^2 = R^2$. By transforming $\iiint_{R}dV$ to spherical coordinates, evaluate the volume of the sphere.

17. (a) Evaluate $\iiint_{E}dV$, where $E$ is the solid enclosed by the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1.$
   Use the transformation $x = au, \ y = bu, \ z = cu.$
   (b) The Earth is not a perfect sphere; rotation has resulted in a flattening at the poles. So the shape can be approximated by an ellipsoid with $a = b = 6378$ km and $c = 6356$ km. Use part (a) to estimate the volume of the Earth.

18. Evaluate $\iiint_{E}x^2y\,dV$, where $E$ is the solid of Exercise 17(a).