Homework 1
Math 104B - Dr. Evans
UCSD Spring 2004

1. Let n be any positive integer. Show that

$$\sum_{j=0}^{n-1} y^j = \frac{y^n - 1}{y - 1}$$

2. Let p be an odd prime. Show that

$$\sum_{j=0}^{p-1} \zeta_p^{j^2} = \sum_{m=0}^{p-1} \left(\frac{m}{p}\right) \zeta_p^m.$$

Hint: First prove

$$\sum_{j=0}^{p-1} \zeta_p^{j^2} = \sum_{m=0}^{p-1} \left[1 + \left(\frac{m}{p}\right)\right] \zeta_p^m$$

3. Let p be a prime such that $p \equiv 3 \pmod 4$. Show that the Gauss sum

$$\sum_{m=0}^{p-1} \left(\frac{m}{p}\right) \zeta_p^m$$

is purely imaginary.