

HOMEWORK 2
Math 104B - Dr. Evans
UCSD Spring 2004

1. Find the 5-adic expansion of -99.
2. Problem 4 from the book: *If $x = a_0 + a_1p + a_2p^2 + \dots$, what is $-x$?*
3. Problem 18 from the book: *Suppose p is an odd prime. Show that if $x^2 \equiv m \pmod{p}$ has a solution and $p \nmid m$, then $x^2 \equiv m \pmod{p^n}$ has a solution for all $n \geq 1$. What happens if $p = 2$?*
4. Let $p = 7$. The previous problem says that there is a p -adic number

$$x = a_0 + a_1p + a_2p^2 + \dots$$

that satisfies the equation

$$a_0 + a_1p + \dots + a_np^n \equiv 2 \pmod{p^{n+1}}$$

for all n .

- (a) Use the technique from the previous problem to explicitly calculate the first three p -adic digits a_0 , a_1 , and a_2 of the solution to the equation $x^2 = 2$. You may assume $a_0 = 4$ (so you really only need to calculate two digits).
 - (b) Plugging in $p = 7$ and the digits found in part a to get the integers $N = a_0 + a_1p + a_2p^2$. Show by direct calculation that $N^2 - 2$ is divisible by $7^3 = 343$. This shows that N is “close to” the square root of 2 in \mathbb{Z}_7 .
5. Find the first four 2-adic digits for the solutions to $x^2 + x + 2$ in \mathbb{Z}_2 .
 6. Let $p = 5$. Consider the p -adic integers

$$\begin{aligned} a &= 3 + 3p + 3p^2 + 3p^3 + 3p^4 + \dots \\ b &= 1 + 2p + 3p^2 + 4p^3 + 0p^4 + \dots \end{aligned}$$

For the following problems, do not assume that a and b are rational numbers. By direct calculation, do the following:

- (a) Find the first five p -adic digits of $a + b$ and $a - b$.
- (b) Find the first three p -adic digits of $a \cdot b$ and $\frac{a}{b}$.

Reading Assignment If you haven't already, you should read Chapter 1 out of Gouvea's book. Do any problems that are not obvious to you or seem interesting. You might also want to start looking at Chapter 2, which contains some more advanced topics.