Challenge Problems 1

Math 104 - Dr. Evans
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1. Pierre Fermat was one of the great early number theorist. All of the statements he claimed to have proved were eventually were eventually proved by mathematicians such as Euler and Gauss, except his famous Last Theorem ($x^n + y^n = z^n$ has no positive integer solution if $n \geq 3$), which was finally proven in 1995 by Andrew Wiles.

This problem involves a conjecture that he made. Fermat thought that all numbers of the form $2^{2^n} + 1$ are prime. It's not difficult to prove that this statement is true for $n = 0$, 1, 2, 3, and 4. Disprove Fermat's conjecture by showing that $2^{2^5} + 1$ is divisible by 641. For extra challenge, do this by hand as Euler did.

2. Using the fact that $1001 = 7 \cdot 11 \cdot 13$, find a divisibility test for 7 based on the digits of the number. Show that the same test (with a slight change) works for divisibility by 13. Can you find a divisibility test for 37?