HOMEWORK 5
Math 104 - Dr. Evans
UCSD Winter 2004
Due Thursday, February 12

1. In class, it was shown that -2 is a square modulo p when $p \equiv 3 \pmod 8$, and -2 is a nonsquare modulo p when $p \equiv 7 \pmod 8$. Prove that -2 is a square modulo p when $p \equiv 1 \pmod 8$, and -2 is a nonsquare modulo p when $p \equiv 5 \pmod 8$.
   (Hint: Separate out the odd factors from the even factors in $(p-1)!$ and apply Wilson's Theorem.)

2. Use the Law of Quadratic Reciprocity to prove that -3 is a square modulo p if and only if $p \equiv 1 \pmod 3$.

3. Use problem 2 and the condition for -1 to be a square modulo p to find a congruence relation on p that is true if and only if 3 is a square modulo p.

4. In class, it was shown that 5 is a square modulo p if and only if $p \equiv \pm 1 \pmod 5$. Use this and the condition for -1 to be a square modulo p to finish the proof that -5 is a square modulo p if and only if $p \equiv 1, 3, 7, \text{ or } 9 \pmod{20}$.